AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 21 Fall 2021

CAPE and CIN

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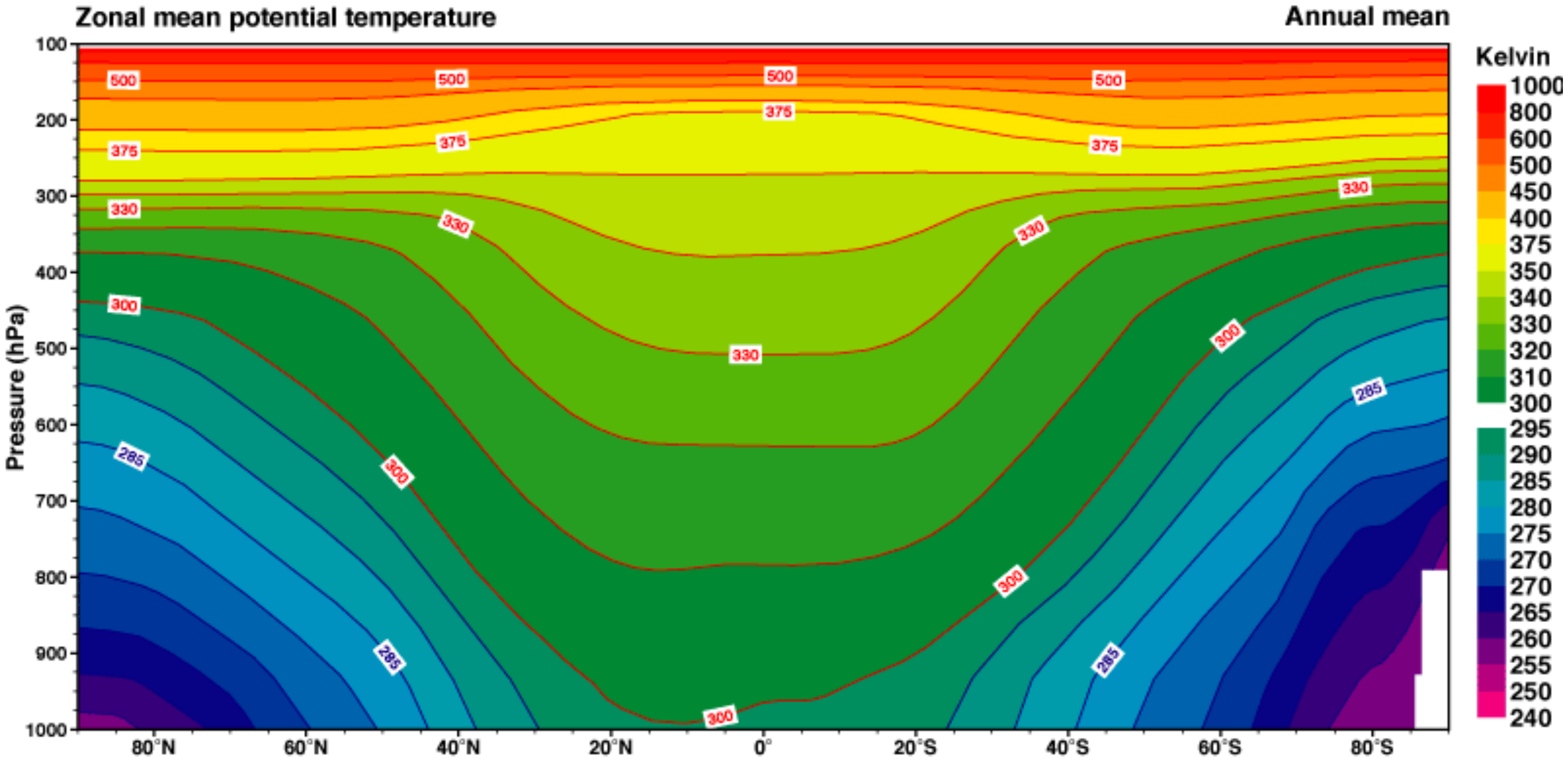
Announcements

HW4 is due Today. Please upload to Canvas by the end of the day.

HW5 has been uploaded to Canvas. It is due in two weeks.

Last class: Stability solutions

Theta profile	Lapse rate	Stability
$\frac{\partial \theta_e}{\partial z} > 0$	$\Gamma < \Gamma_m$	*Stable*
$\frac{\partial \theta_e}{\partial z} = 0$	$\Gamma = \Gamma_m$	*Moist Neutral*
$\frac{\partial \theta_e}{\partial z} < 0$	$\Gamma_m < \Gamma < \Gamma_d$	*Conditionally Unstable*
$\frac{\partial \theta}{\partial z} = 0$	$\Gamma = \Gamma_d$	*Dry Neutral*
$\frac{\partial \theta}{\partial z} > 0$	$\Gamma > \Gamma_d$	*Absolutely Unstable*



The troposphere is usually stable to dry motions, but can be unstable to saturated motions.

Available potential energy

In classical mechanics, total energy (TE) is the sum of potential and kinetic energies (PE & KE).

$$TE = KE + PE$$

Energy is a conserved quantity

$$\frac{D}{Dt}(KE) = -\frac{D}{Dt}(PE)$$

How does this relate to buoyancy?

$$\frac{Dw}{Dt} = B$$

Available potential energy

For a 1-D parcel rising through the troposphere

$$w\frac{Dw}{Dz} = B$$

Can rewrite as

$$\frac{D}{Dz}\left(\frac{w^2}{2}\right) = B$$

Where the left hand side describes the change in KE as the parcel rises. By definition, buoyancy must be the change in PE as the parcel rises!

$$B = -\frac{D}{Dz}(APE)$$

Available potential energy

$$B = -\frac{D}{Dz}(APE)$$

Where APE means **Available Potential Energy.**

It is defined this way since **most potential energy in our atmosphere cannot be readily converted to kinetic energy.** This is due to the strong constraint of hydrostatic balance and our planet's rotation.

Available potential energy

Can integrate equation to obtain the conversion of PE to KE

$$-\Delta APE = \int_{z_1}^{z_2} B dz$$

Is the change in available potential energy

We generally split the integral into components

$$-\Delta APE = \Delta APE(B > 0) - \Delta APE(B < 0)$$

We define the **Level of Free Convection** as the height where the parcel becomes buoyant (B >0)

$$LFC = z(B > 0)$$

Available potential energy

Can integrate equation to obtain the conversion of PE to KE

$$-\Delta APE = \int_{z_1}^{z_2} B dz$$

Is the change in available potential energy

We generally split the integral into components $-\Delta APE = CAPE - CIN$

$$\Delta KE = CAPE - CIN$$

$$CAPE = \int_{LFC}^{LZB} B dz$$

Is the Convective Available Potential Energy

$$CIN = -\int_{z_1}^{LFC} B dz$$

Is the Convective Inhibition

Why CAPE and CIN?

$$CIN = -\int_{z_1}^{LFC} B dz$$

The Convective Inhibition is the intregrated region where B < 0.

Work must be done to lift a parcel over this region (e.g. by a front)

CIN is the work that must be done to lift a parcel from a height z1 (usually the surface) up to the region where it becomes buoyant.

Why CAPE and CIN?

$$CAPE = \int_{LFC}^{LZB} B dz$$

The CAPE integrates the region of troposphere where B > 0.

The level of free convection is when a saturated parcel becomes buoyant

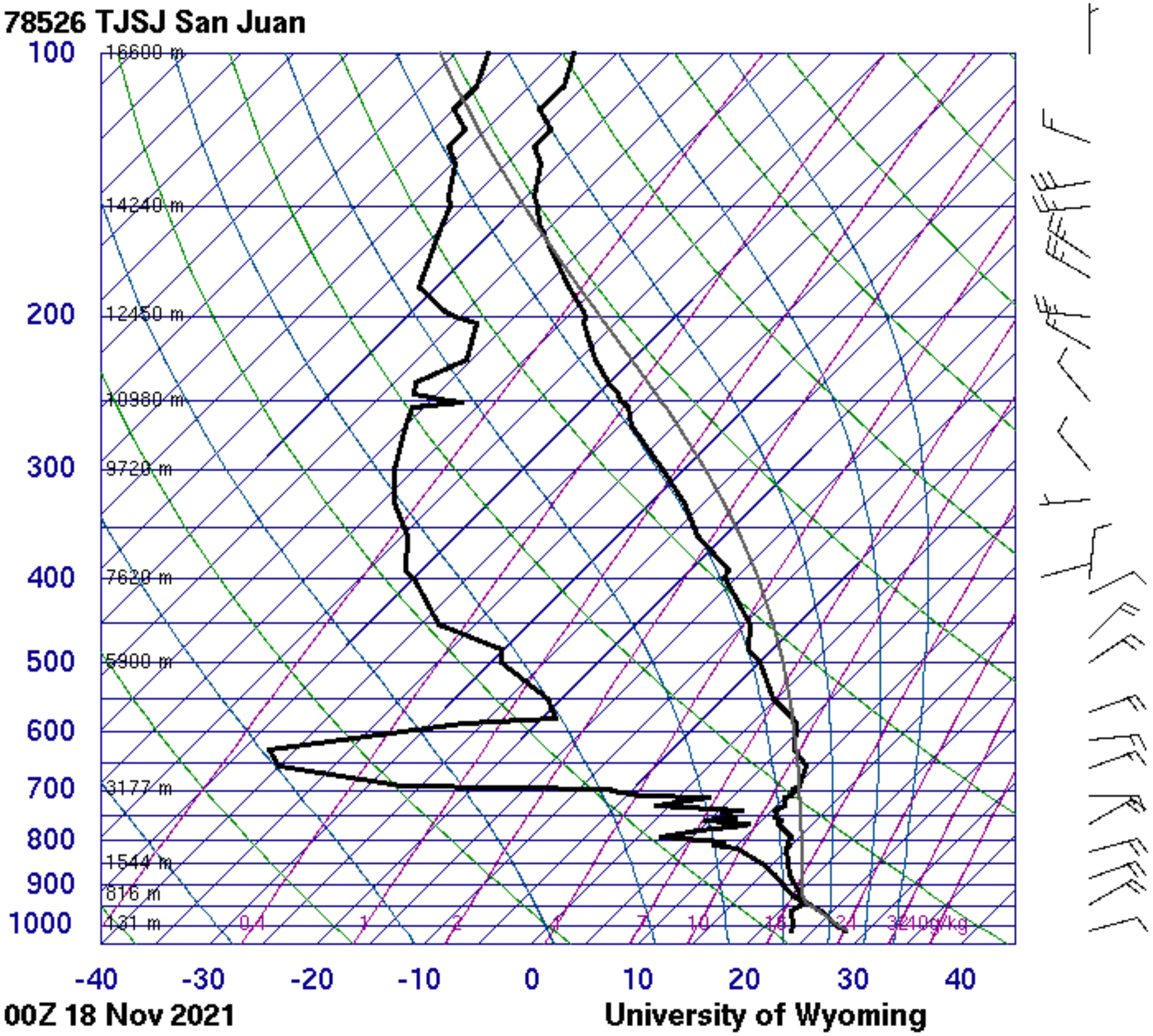
CAPE is the integrated buoyancy from the level of free convection until the level of zero buoyancy, where B becomes negative again

Updraft velocity

Cal also calculate a parcel's maximum vertical velocity based on the kinetic energy change. Assuming the parcel starts approximately at rest.

$$w_{max} = \sqrt{2CAPE}$$

Exercise



Assume that a parcel is lifted from the surface. Based on this sounding find:

1. The mixing ratio at the surface.
2. The LCL
3. The areas where B>0 (CAPE)
4. The areas where B<0 (CIN)
5. The level of neutral buoyancy.

Caveats about CAPE and Wmax

The calculation of CAPE and w_{max} assumes

- No aerodynamic drag
- No effects of the mass of liquid or ice in the parcel (HW5 last problem)
- No mixing

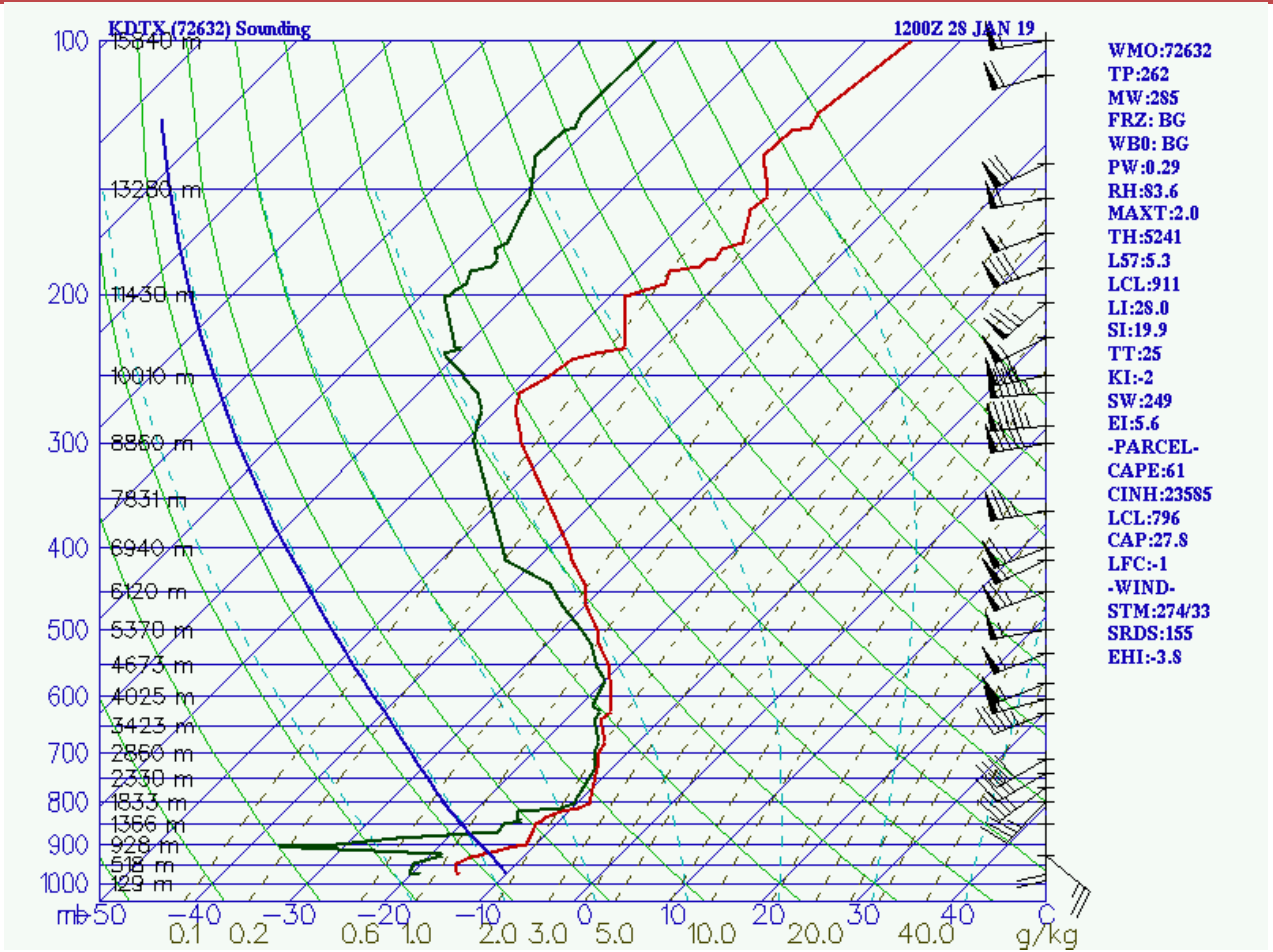
Because of this, wmax is an overestimate of the actual vertical velocities observed in deep convection

Mechanically-forced lifting

In class, we've seen that ascent can be triggered by buoyancy

But what if the sounding does not support positive buoyancy?

$$\frac{Dw}{Dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + B$$



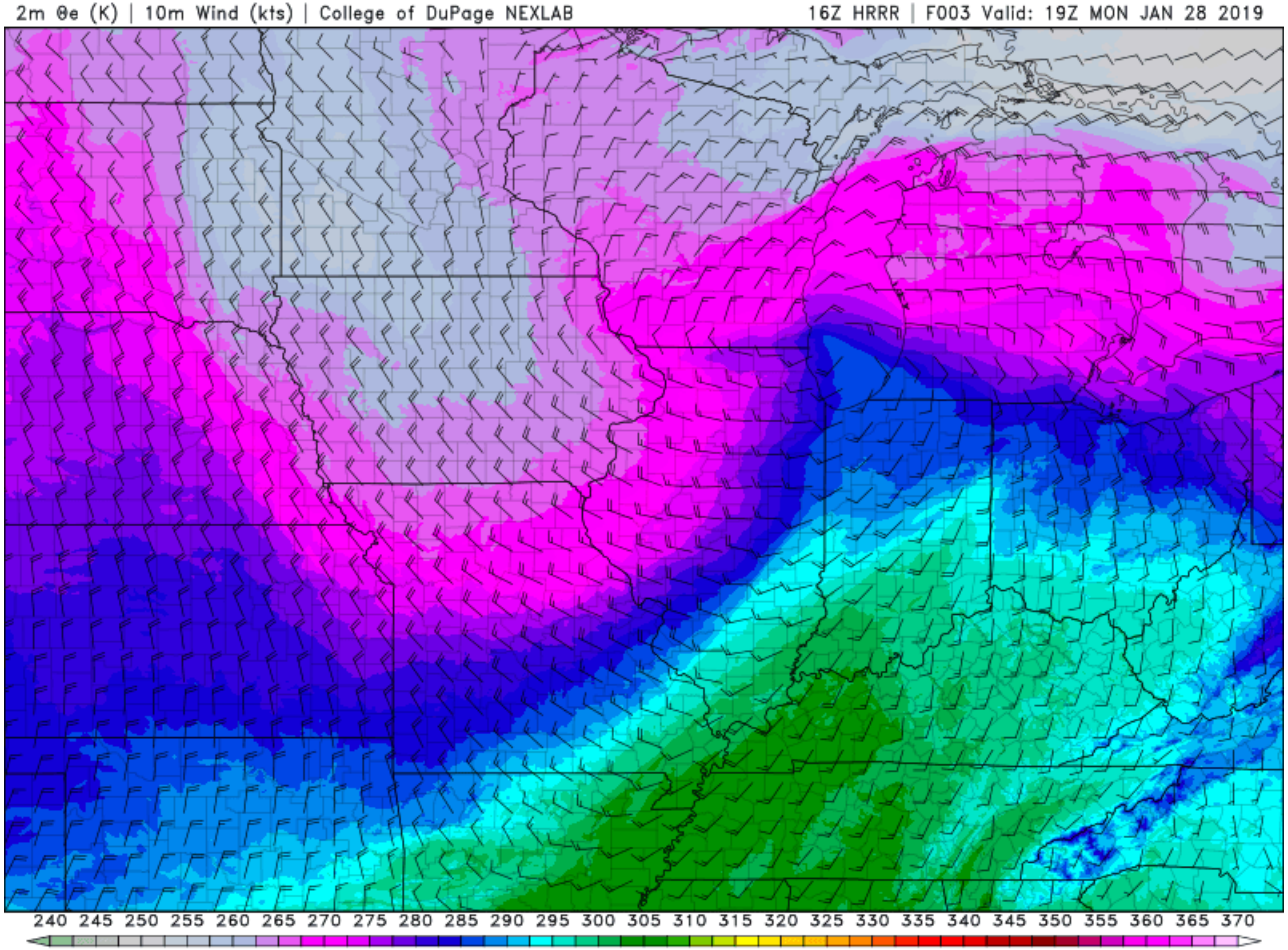
Synoptic-scale (~1000 km) ascent

For large storm systems, vertical acceleration is weak.

Buoyancy and pressure gradients cancel out and hydrostatic balance once again emerges.

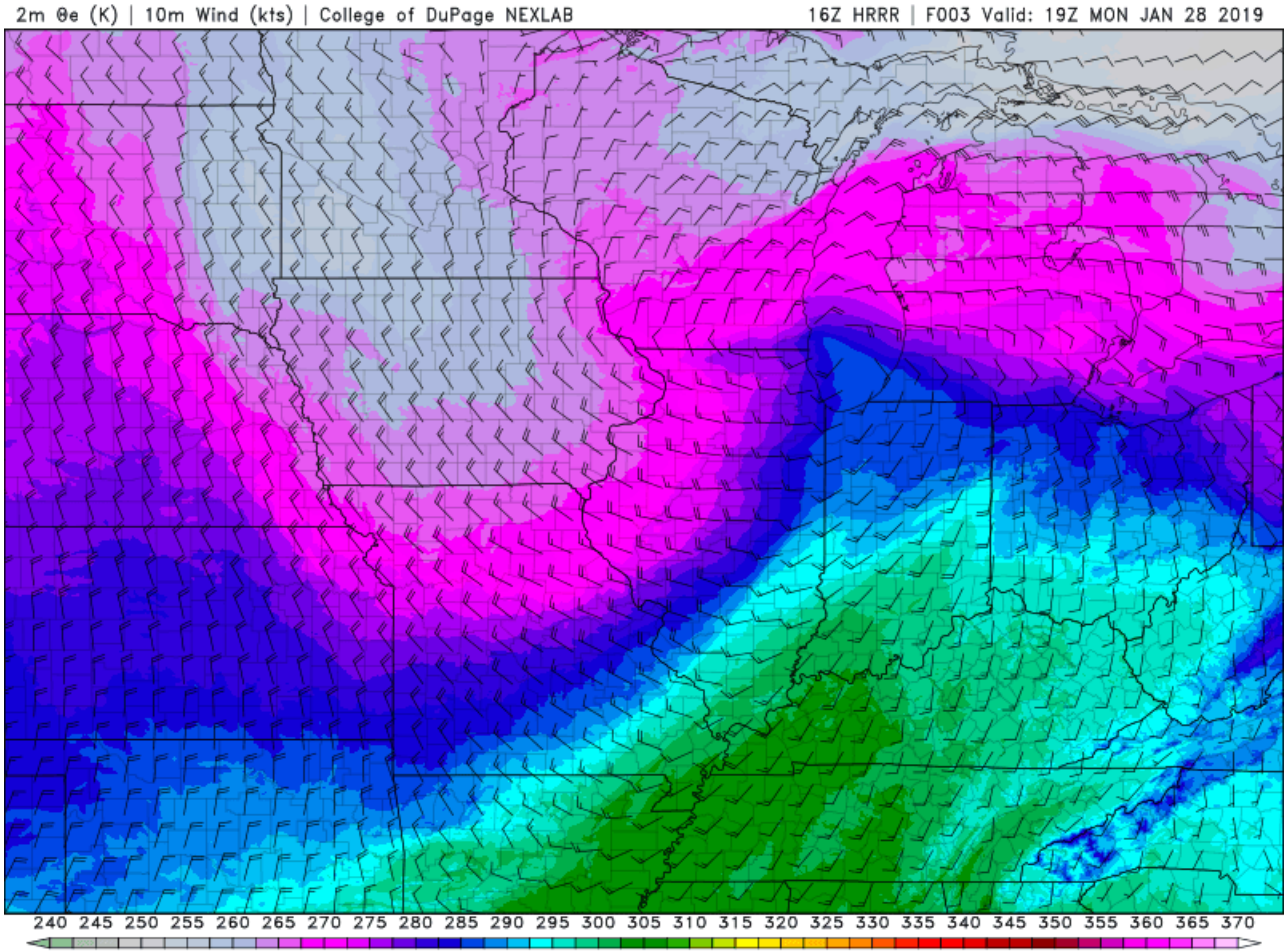
$$\frac{1}{\rho_0}\frac{\partial p'}{\partial z} \simeq B$$

Resulting vertical motion is weak



Synoptic-scale (~1000 km) ascent

Notice however, the strong gradient in equivalent potential temperature.



Entropy and vertical motion

$$\frac{D\theta_e}{Dt} \simeq 0$$

Equivalent potential temperature is conserved

Can expand using the chain rule to obtain

$$\frac{\partial\theta_e}{\partial t} + u\frac{\partial\theta_e}{\partial x} + v\frac{\partial\theta_e}{\partial y} + w\frac{\partial\theta_e}{\partial z} \simeq 0$$

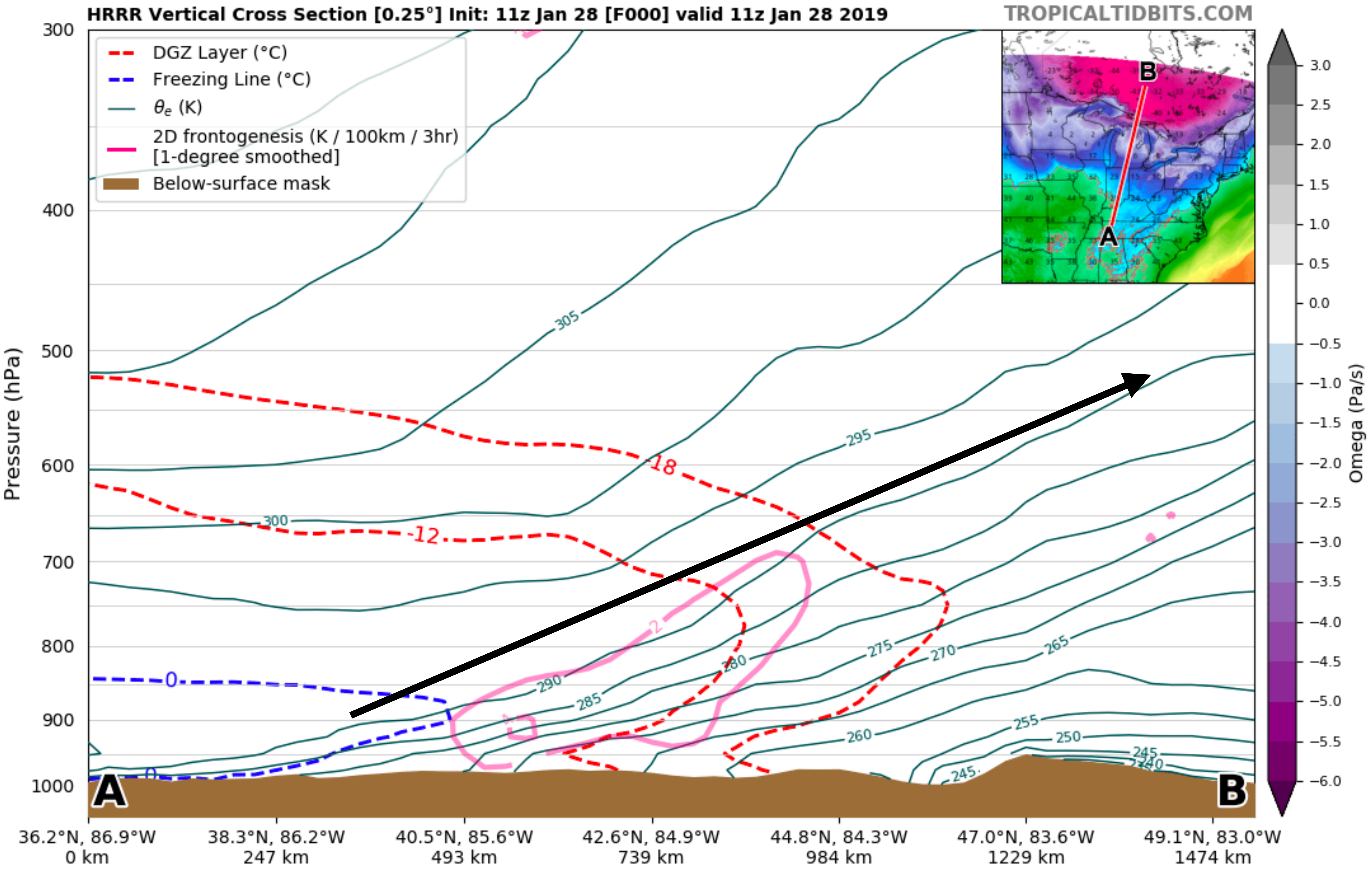
By looking at the previous map we can guess that, locally

$$w\frac{\partial\theta_e}{\partial z} = -v\frac{\partial\theta_e}{\partial y}$$

Mechanically-forced lifting

$$w\frac{\partial\theta_e}{\partial z} = -v\frac{\partial\theta_e}{\partial y}$$

Ascent occurs along lines of constant theta-e



How fast is air rising?

$$w = -v\frac{\partial \theta_e}{\partial y}\left(\frac{\partial \theta_e}{\partial z}\right)^{-1}$$

$$v \sim 10m/s$$

Estimating the gradients based on cross section yields

$$w \sim 10cm/s$$