AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 20   Fall 2021

Stability

Ángel F. Adames-Corraliza
angel.adamescorraliza@wisc.edu

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Announcements

Please upload your Skew-Ts whenever you have the chance.

No Skew-T a week this week. Instead, you will have HW5 problems with Skew-Ts

HW4 is due on Thursday.

No class next week.

Defining buoyancy

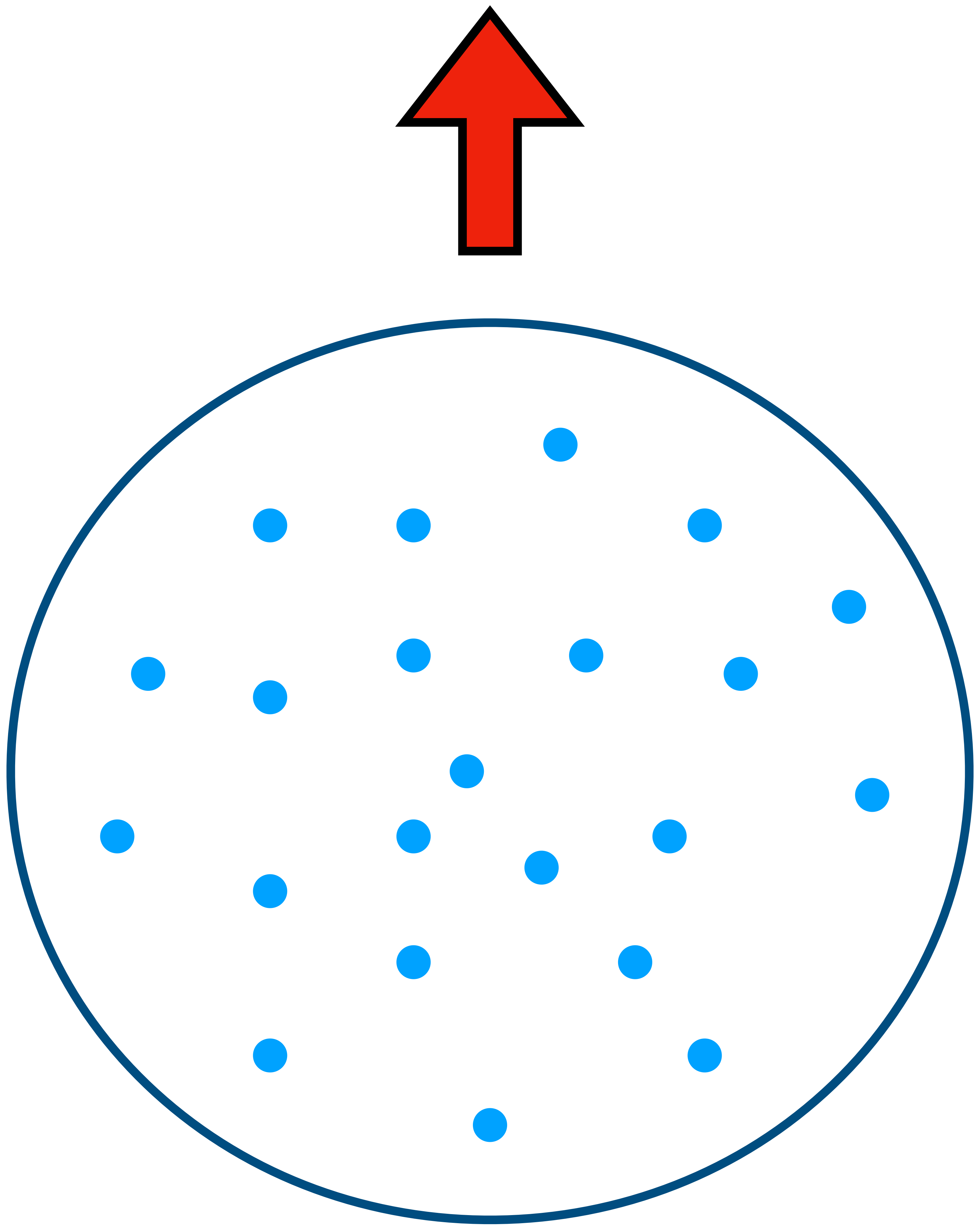
$$\frac{Dw}{Dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + B$$

Perturbation Pressure gradient force

Buoyancy

$$B \equiv -g\frac{\rho - \rho_0}{\rho_0}$$

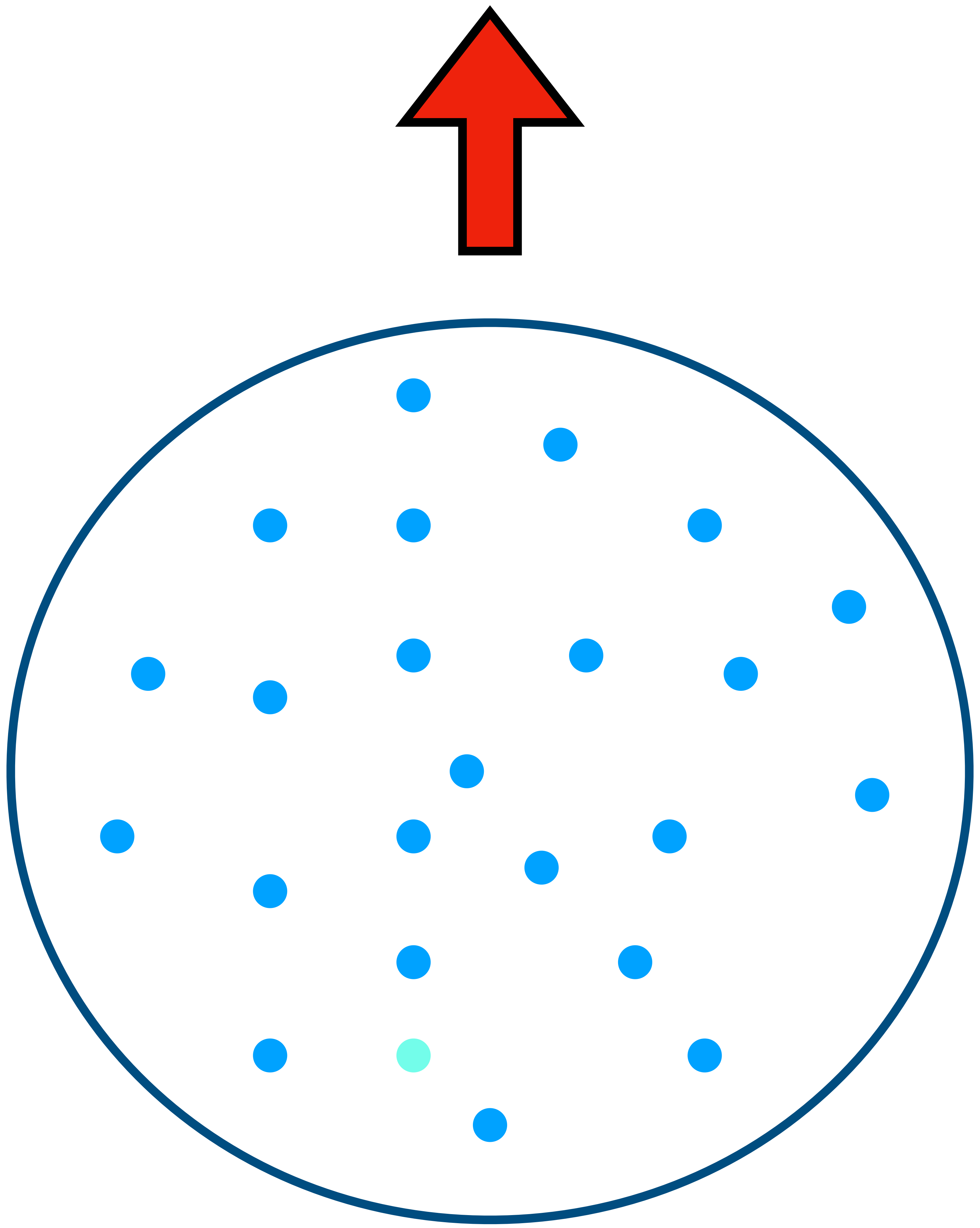
How can a parcel accelerate upward?



For moist unsaturated air

Can express the buoyancy as the difference in virtual temperature between the parcel and its surroundings.

$$B \simeq g \frac{T_v - T_{v0}}{T_{v0}}$$

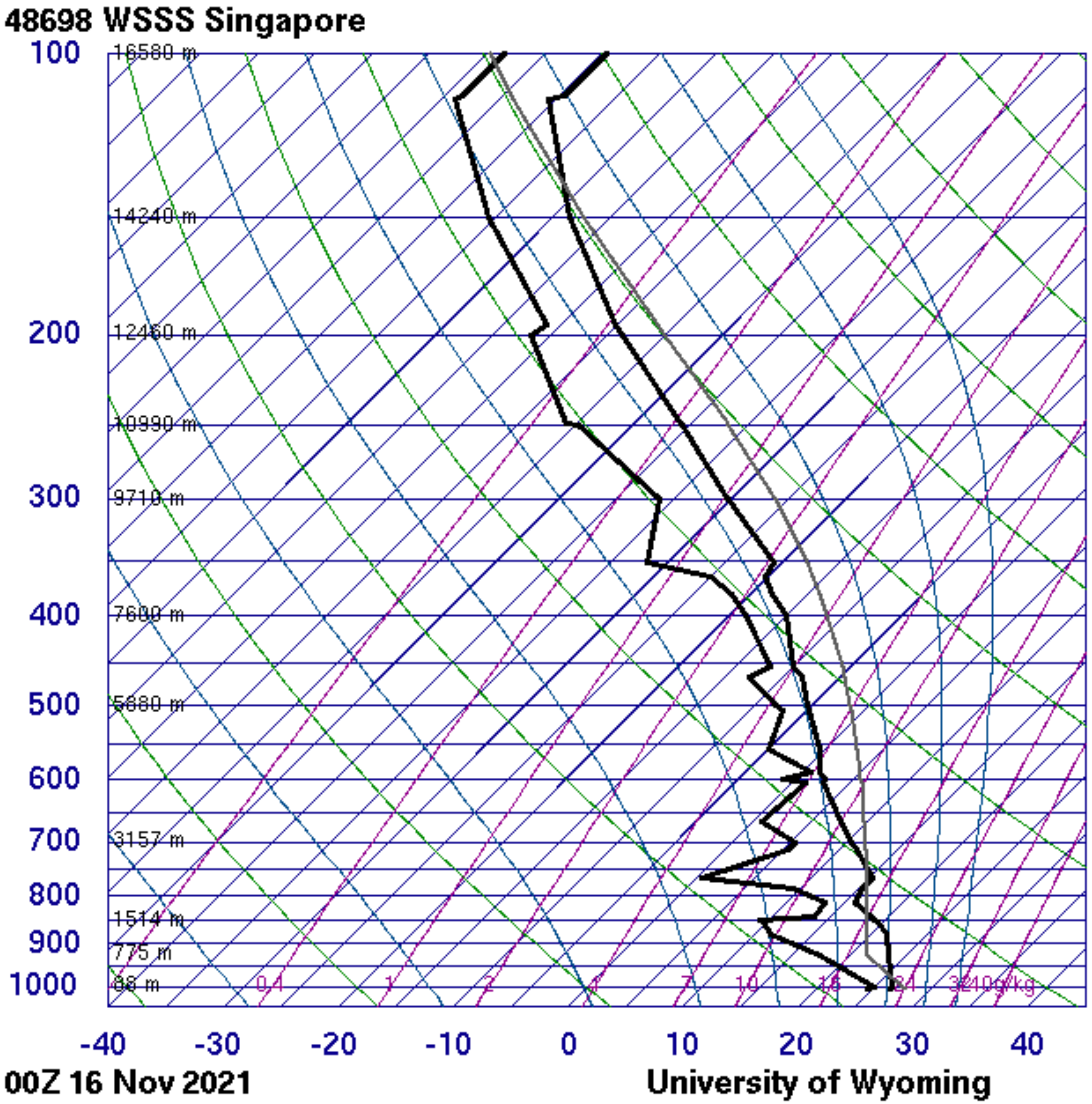


For moist unsaturated air

Outside hot humid regions, the virtual effect can be ignored

$$B \simeq g\frac{T - T_0}{T_0}$$

On a Skew-T, buoyant parcels will have temperatures to the right of the observed one.



Ignoring perturbations in the pressure gradient force, our acceleration becomes

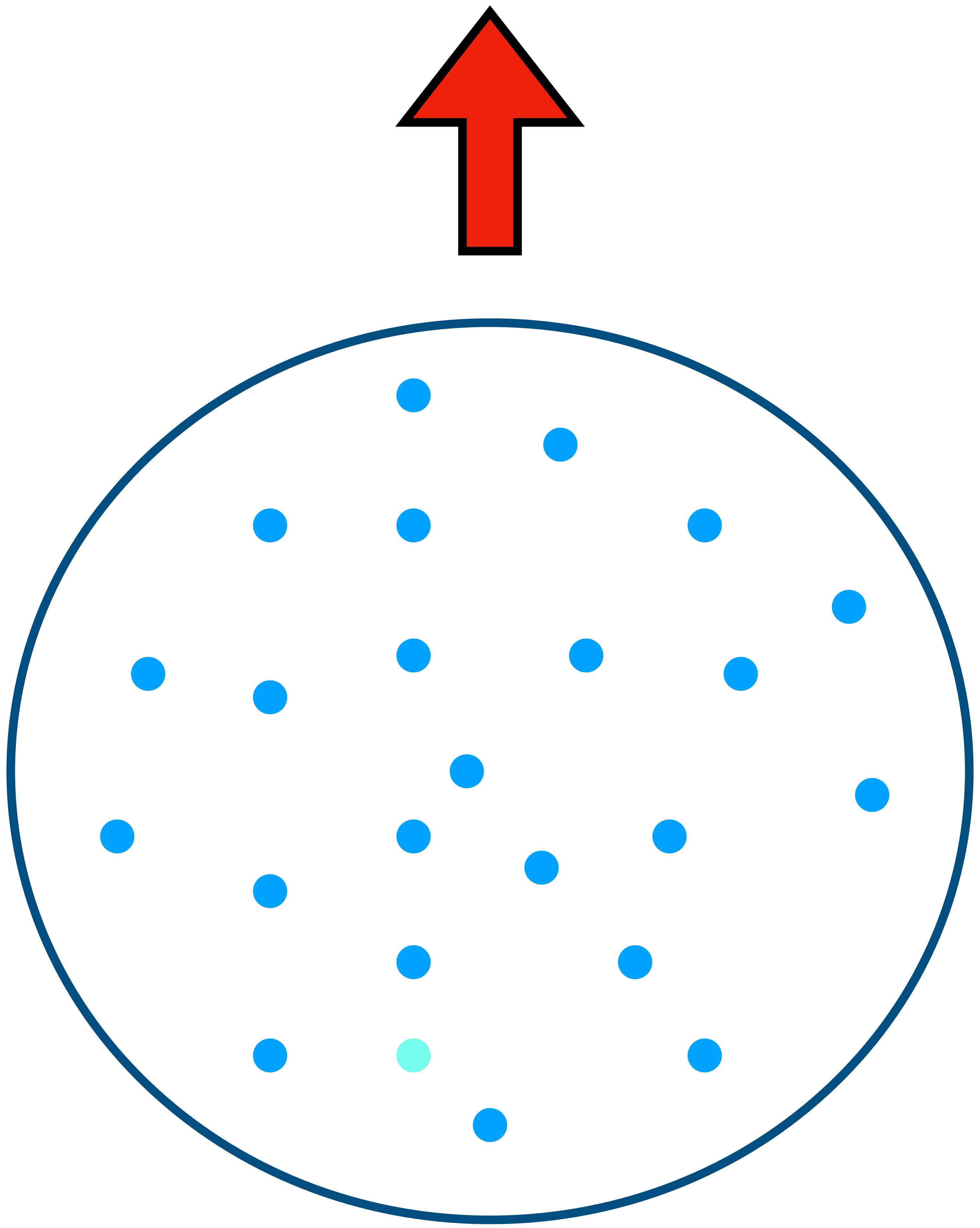
$$\frac{Dw}{Dt} = g\frac{T - T_0}{T_0}$$

Recognizing that the vertical velocity is just the change of height wit time

$$w = \frac{Dz}{Dt}$$

We get

$$\frac{D^2 z}{Dt^2} = g\frac{T - T_0}{T_0}$$



Atmospheric Stability

For a dry atmosphere

$$\frac{D^2 z}{Dt^2} = g\frac{T - T_0}{T_0}$$

For our planet's troposphere, temperatures change linearly with height

$$T(z) = T_s - \Gamma_d z$$

$$T_0(z) = T_s - \Gamma z$$

Dry adiabatic lapse rate

$$\Gamma_d = \frac{g}{c_p}$$

Environmental Lapse rate

$$\Gamma = -\frac{\partial T}{\partial z}$$

For a dry atmosphere

$$\frac{D^2 z}{Dt^2} = g\frac{(\Gamma - \Gamma_d)z}{T_0}$$

We can further simplify by using the potential temperature definition

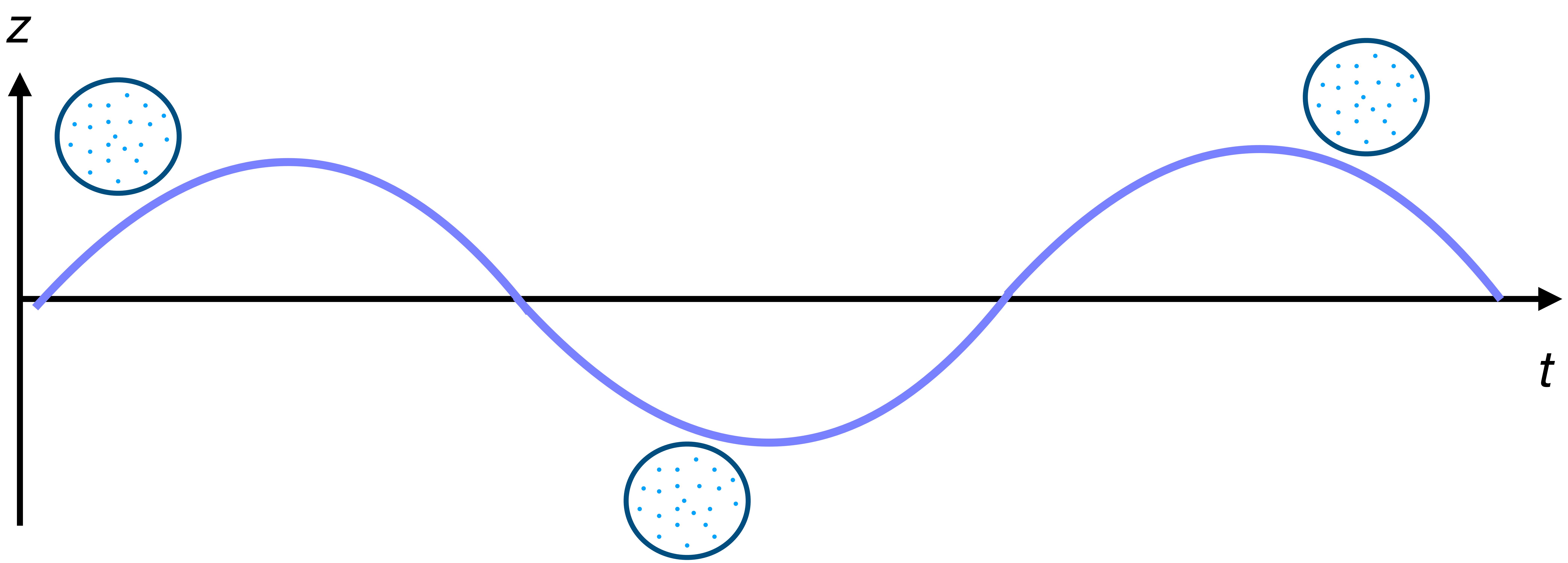
$$\frac{1}{\theta}\frac{\partial \theta}{\partial z} = \frac{1}{T}\left(\Gamma_d - \Gamma\right)$$

$$\frac{D^2 z}{Dt^2} = -N^2 z$$

$$N^2 = \frac{g}{\theta}\frac{\partial \theta}{\partial z}$$

Atmospheric Stability: Solutions

BV freq.	Stratification	Stability	ODE Solution (N constant)
$N > 0$	$\frac{\partial \theta}{\partial z} > 0$	*Stable*	$z(t) = c_1 \cos(Nt) + c2 \sin(Nt)$



Can a stable atmosphere create clouds?

Can a stable atmosphere create clouds?

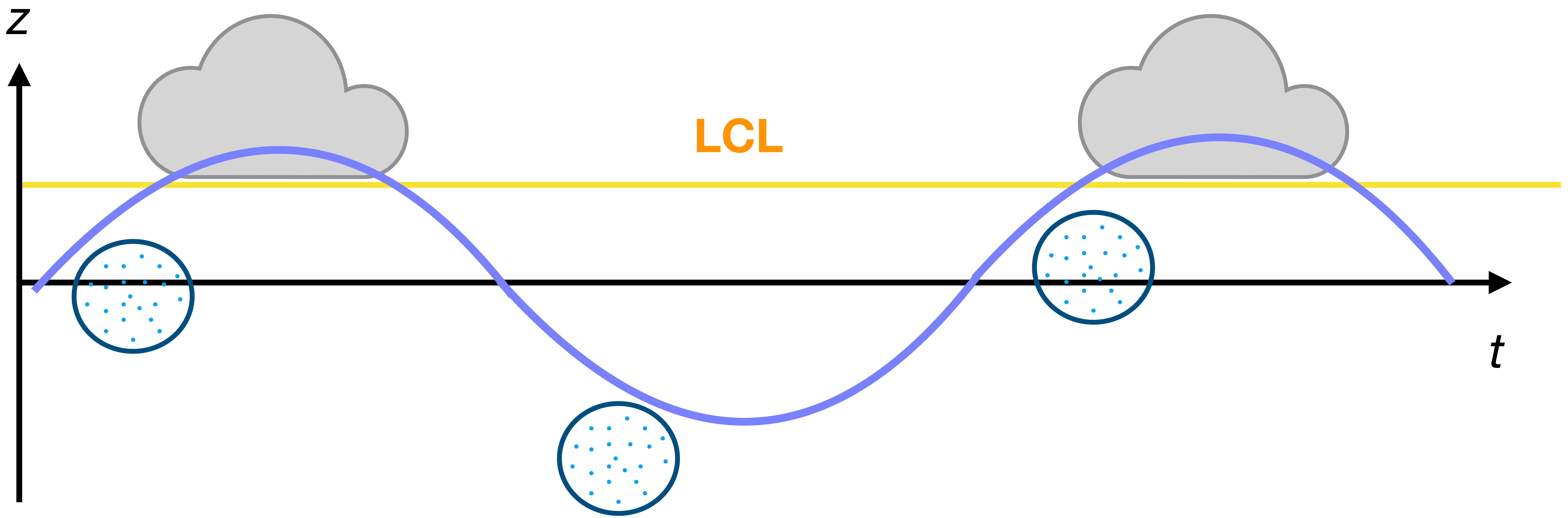
Lifting in stable atmospheres can create clouds only if the lifting is already close to the cloud layer.

Cooling and other processes can create clouds (e.g. fog) even if the atmosphere is stable.

Can a stable atmosphere create clouds?

Lifting in stable atmospheres can create clouds only if the lifting is already close to the cloud layer.

Cooling and other processes can create clouds (e.g. fog) even if the atmosphere is stable.

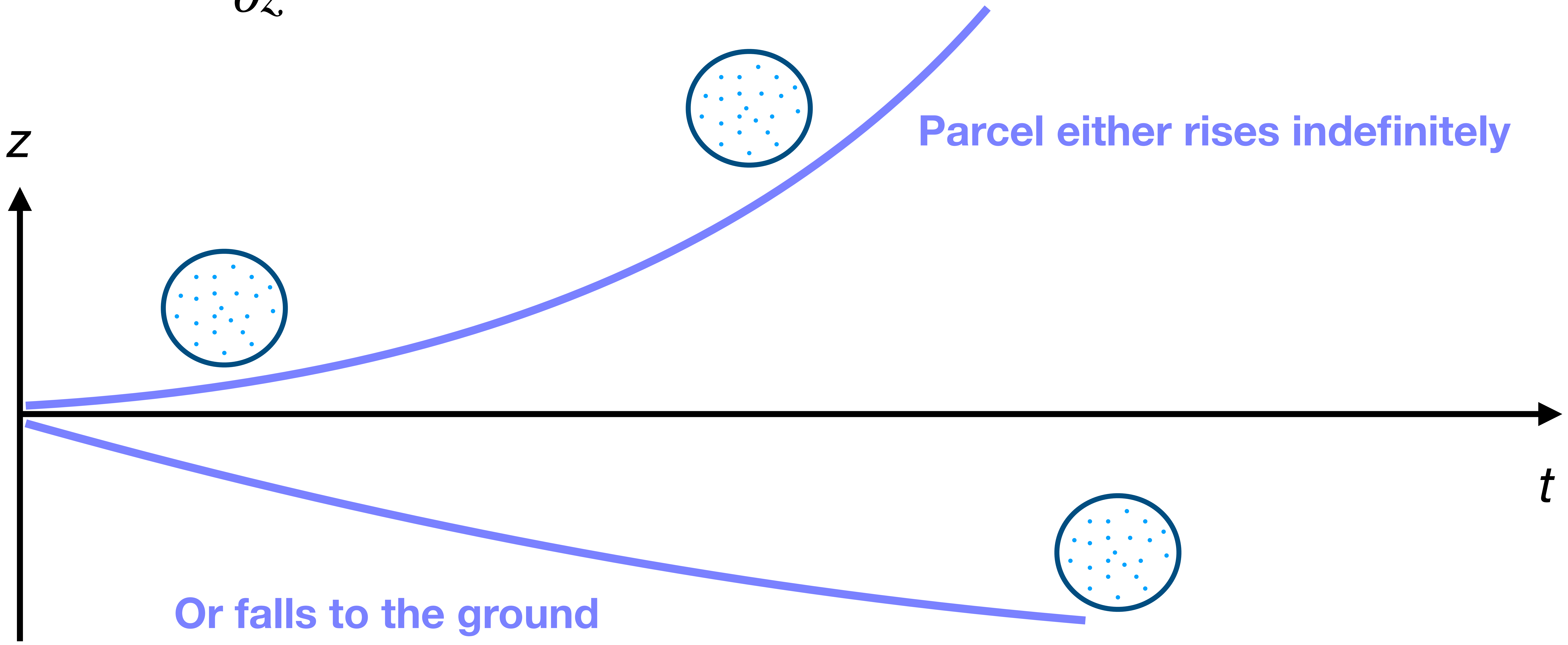


Can a stable atmosphere create clouds?

Can it create deep cumulonimbus clouds though?

Atmospheric Stability: Solutions

BV freq.	Stratification	Stability	ODE Solution (N constant)
$N^2 < 0$	$\frac{\partial\theta}{\partial z} < 0$	*Unstable*	$z(t) = c_1 \exp(Nt) + c2 \exp(-Nt)$

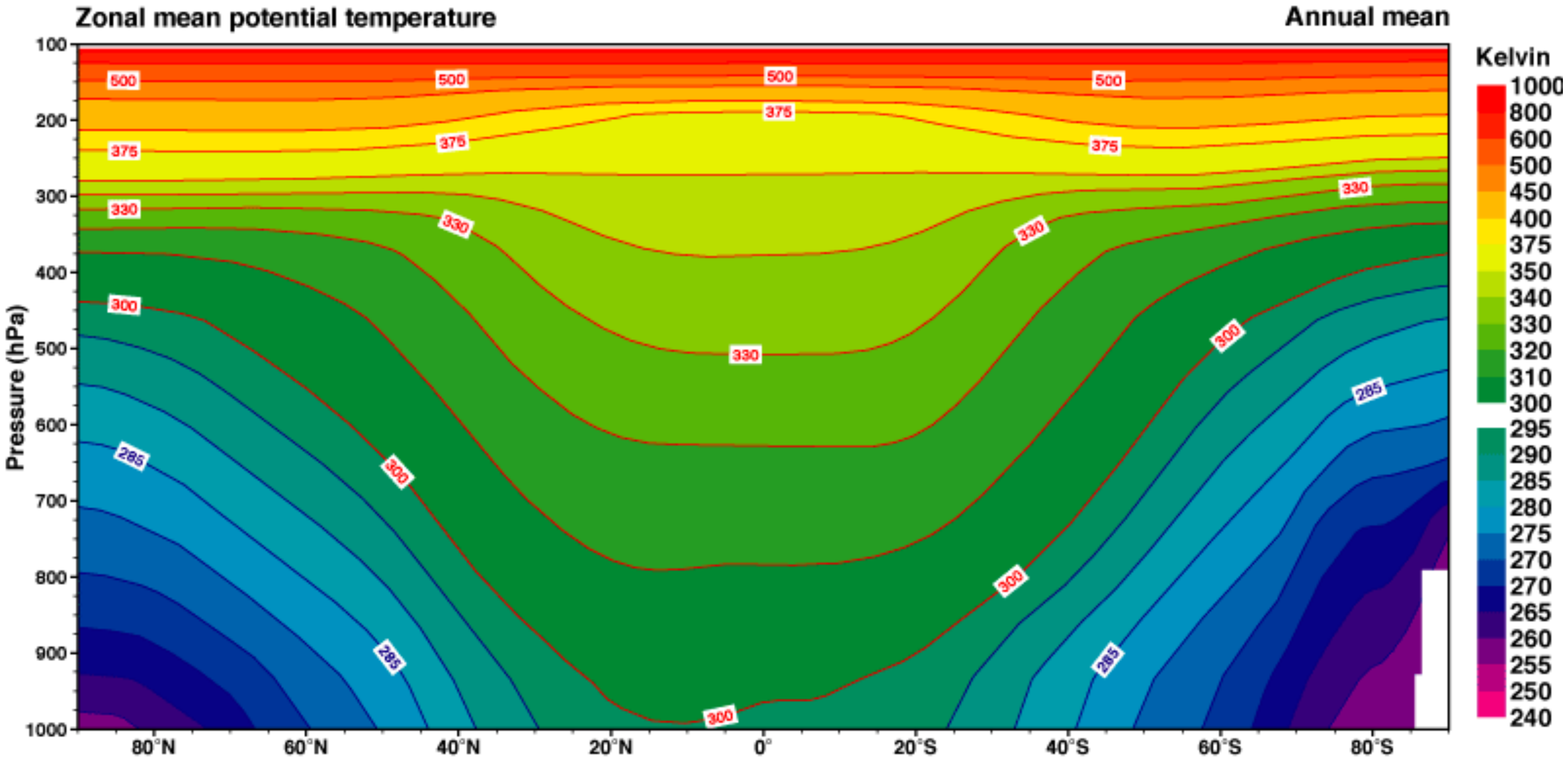


Parcel either rises indefinitely

Or falls to the ground

Can an unstable atmosphere create clouds?

Is the atmosphere usually stable or unstable?



Dry Stability

The atmosphere is usually statically stable.

It is only unstable under specific conditions, such as when the ground warms due to solar heating. This only happens near the surface.

Need other processes to account for the development of most deep clouds.

Atmospheric Stability: Moist case

For a saturated atmospheric layer

$$\frac{D^2 z}{Dt^2} = g\frac{T - T_0}{T_0}$$

$$T(z) = T_s - \Gamma_m z$$

$$T_0(z) = T_s - \Gamma z$$

Moist adiabatic lapse rate

$$\Gamma_m = \Gamma_d \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}}$$

Environmental Lapse rate

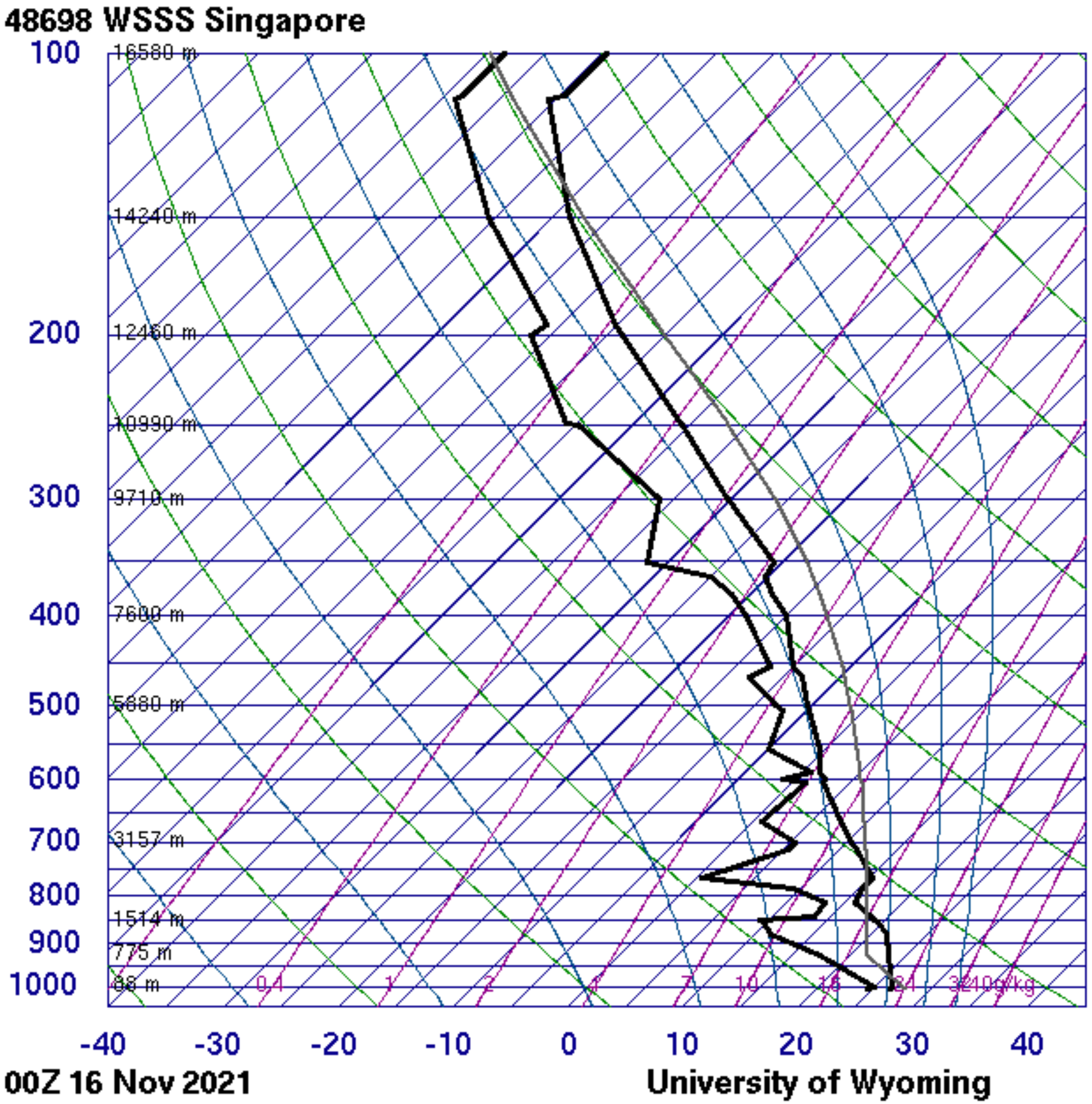
$$\Gamma = -\frac{\partial T}{\partial z}$$

The same solutions arise, but are now with respect to a moist adiabat, which has a smaller value than the dry adiabat.

Atmospheric Stability: Moist case

The atmosphere is usually stable to dry motions, but can be unstable to moist motions.

This is often referred to as **conditional instability**.



Stability solutions

Theta profile	Lapse rate	Stability
$\frac{\partial \theta_e}{\partial z} > 0$	$\Gamma < \Gamma_m$	*Stable*
$\frac{\partial \theta_e}{\partial z} = 0$	$\Gamma = \Gamma_m$	*Moist Neutral*
$\frac{\partial \theta_e}{\partial z} < 0$	$\Gamma_m < \Gamma < \Gamma_d$	*Conditionally Unstable*
$\frac{\partial \theta}{\partial z} = 0$	$\Gamma = \Gamma_d$	*Dry Neutral*
$\frac{\partial \theta}{\partial z} > 0$	$\Gamma > \Gamma_d$	*Absolutely Unstable*

Available potential energy

In classical mechanics, total energy (TE) is the sum of potential and kinetic energies (PE & KE).

$$TE = KE + PE$$

Energy is a conserved quantity

$$\frac{D}{Dt}(KE) = -\frac{D}{Dt}(PE)$$

How does this relate to buoyancy?

$$\frac{Dw}{Dt} = B$$

Available potential energy

For a 1-D parcel rising through the troposphere

$$w\frac{Dw}{Dz} = B$$

Can rewrite as

$$\frac{D}{Dz}\left(\frac{w^2}{2}\right) = B$$

Where the left hand side describes the change in KE as the parcel rises. By definition, buoyancy must be the change in PE as the parcel rises!

$$B = -\frac{D}{Dz}(APE)$$

Available potential energy

$$B = -\frac{D}{Dz}(APE)$$

Where APE means **Available Potential Energy.**

It is defined this way since most potential energy in our atmosphere cannot be readily converted to kinetic energy. This is due to the strong constraint of hydrostatic balance and our planet's rotation.

Available potential energy

Can integrate equation to obtain the conversion of PE to KE

$$-\Delta APE = \int_{z_1}^{z_2} B dz$$

Is the change in available potential energy

We generally split the integral into components

$$-\Delta APE = \Delta APE(B > 0) - \Delta APE(B < 0)$$

We define the **Level of Free Convection** as the height where the parcel becomes buoyant (B >0)

$$LFC = z(B > 0)$$

Available potential energy

Can integrate equation to obtain the conversion of PE to KE

$$-\Delta APE = \int_{z_1}^{z_2} B dz$$

Is the change in available potential energy

We generally split the integral into components $-\Delta APE = CAPE - CIN$

$$\Delta KE = CAPE - CIN$$

$$CAPE = \int_{LFC}^{LZB} B dz$$

Is the Convective Available Potential Energy

$$CIN = -\int_{z_1}^{LFC} B dz$$

Is the Convective Inhibition

Why CAPE and CIN?

$$CIN = -\int_{z_1}^{LFC} B dz$$

The Convective Inhibition is the intregrated region where B < 0.

Work must be done to lift a parcel over this region (e.g. by a front)

CIN is the work that must be done to lift a parcel from a height z1 (usually the surface) up to the region where it becomes buoyant.

Why CAPE and CIN?

$$CAPE = \int_{LFC}^{LZB} B dz$$

The CAPE integrates the region of troposphere where B > 0.

The level of free convection is when a saturated parcel becomes buoyant

CAPE is the integrated buoyancy from the level of free convection until the level of zero buoyancy, where B becomes negative again

Updraft velocity

Cal also calculate a parcel's maximum vertical velocity based on the kinetic energy change. Assuming the parcel starts approximately at rest.

$$w_{max} = \sqrt{2CAPE}$$