AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 17 Fall 2021

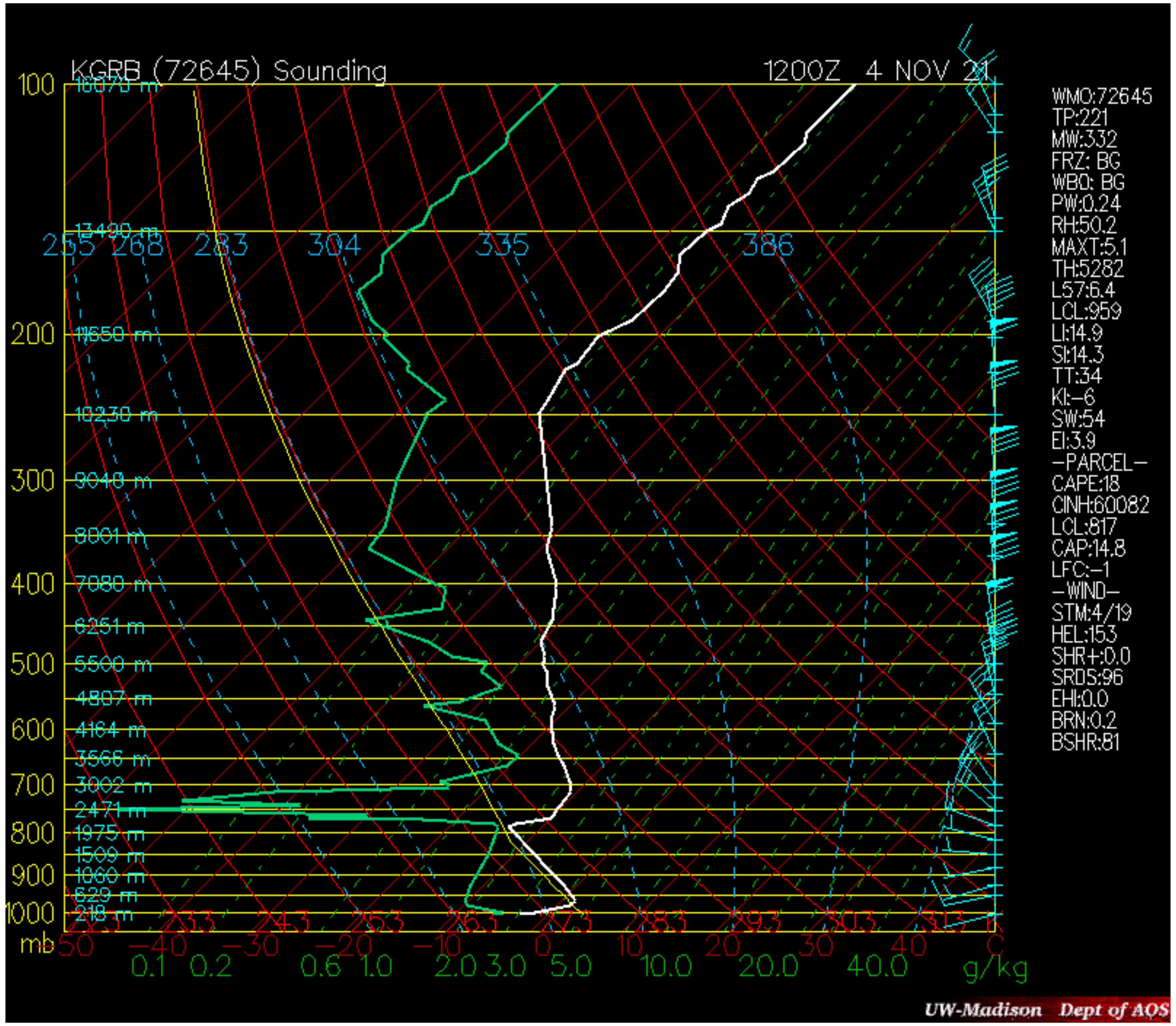
Carnot Engine 2, Buoyancy and convection

Ángel F. Adames-Corraliza
angel.adamescorraliza@wisc.edu

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Announcements

Next Tuesday we will discuss the section titled “**The mature hurricane: A natural Carnot engine**” by Emanuel (1991) (TC_Carnot_Engine.pdf on Canvas).



https://earth.nullschool.net/#2021/08/25/1600Z/wind/isobaric/500hPa/orthographic=-87.74,30.76,1229

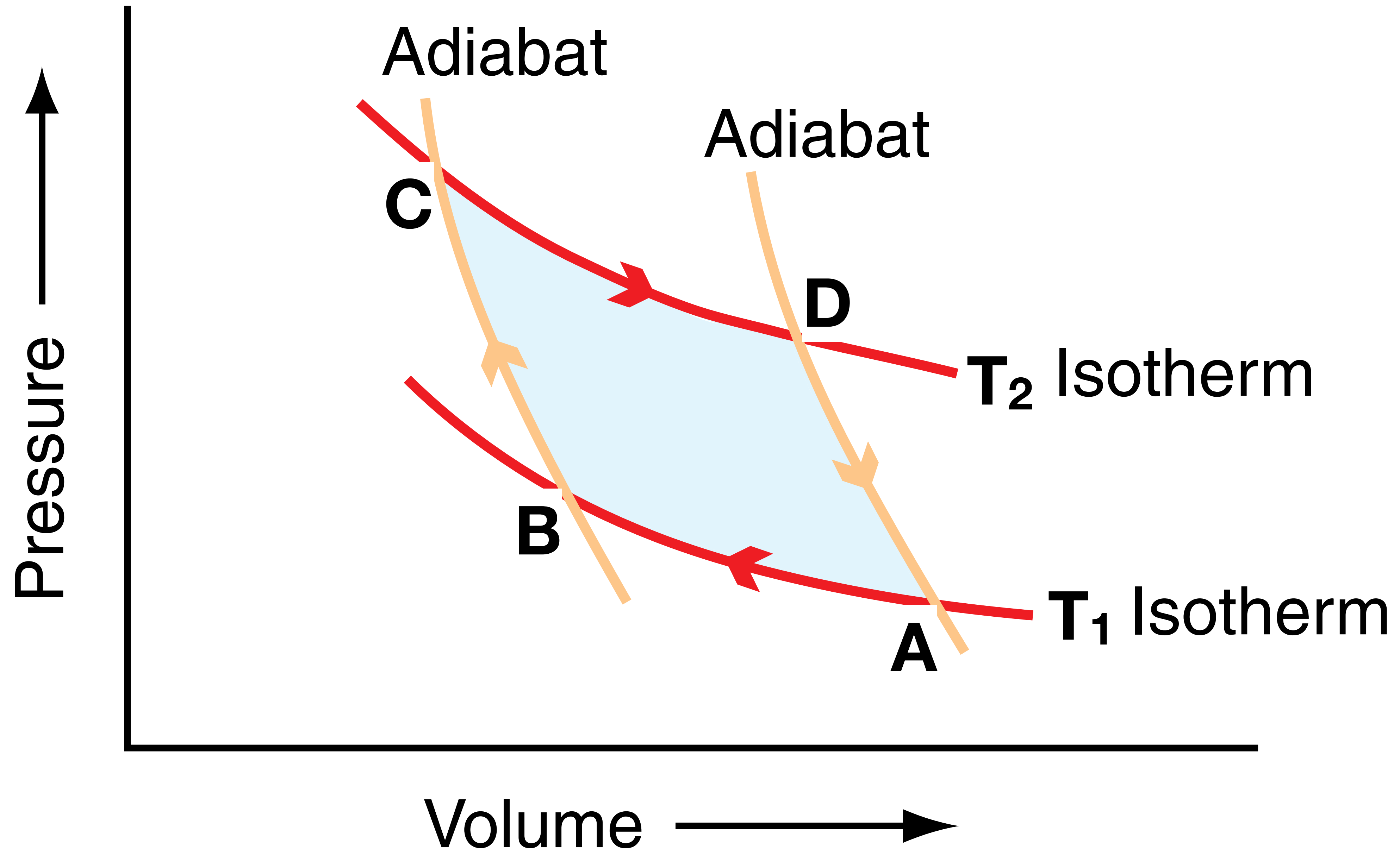
Last Class: Carnot Engine

The first law (internal energy form) integrated over this cycle takes the form:

$$\oint c_v dT = \oint \delta q - \oint \delta w$$

Because state variable don't change during a closed loop integral, it follows that

$$\oint \delta q = \oint \delta w$$



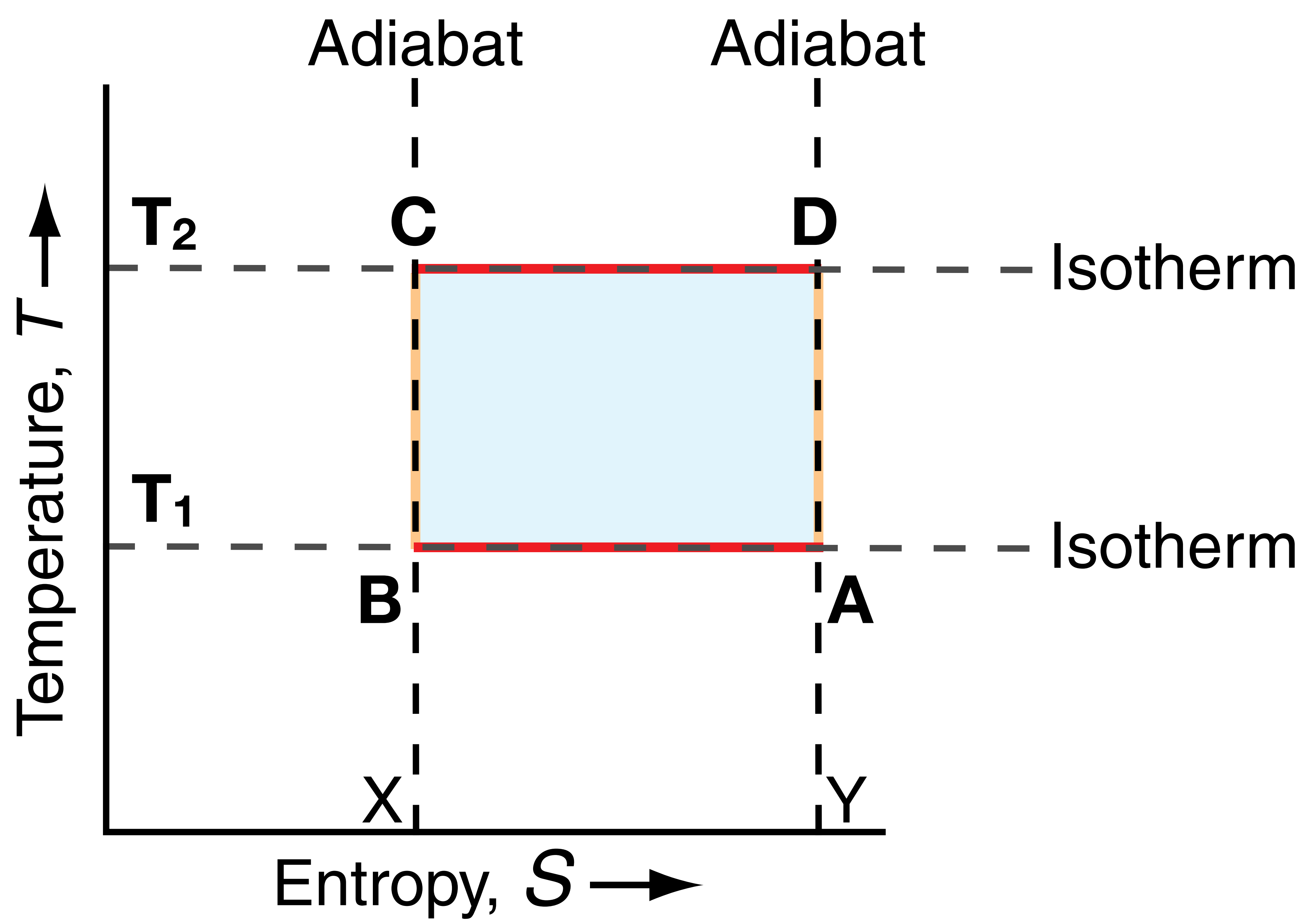
Carnot Engine

Writing these in exact derivative form we have

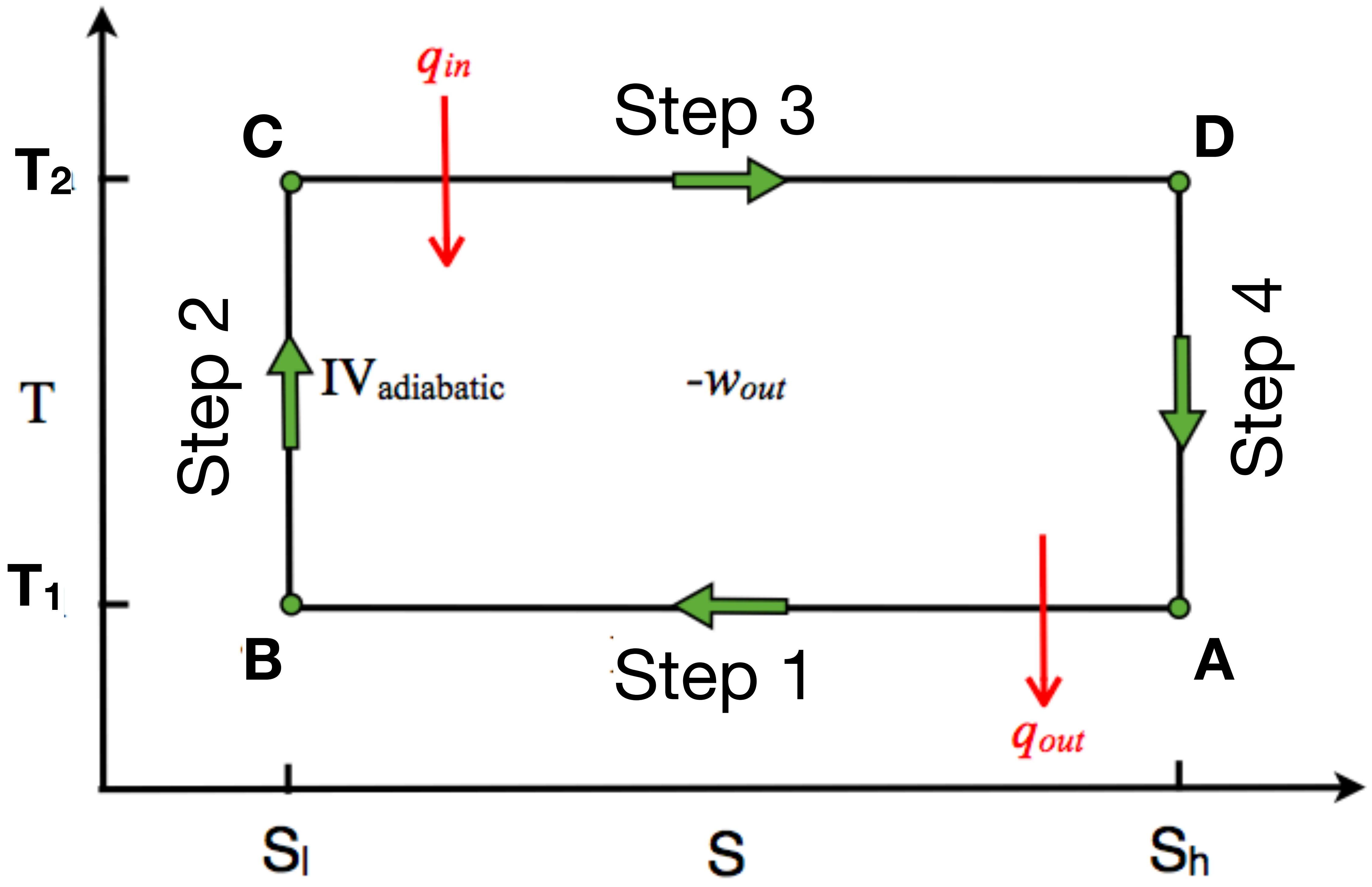
$$\oint T ds = \oint p d\alpha \neq 0$$

Now let's consider a cycle divided into 4 steps:

1. Isothermal compression at a cooler T1
2. Adiabatic compression to T2
3. Isothermal expansion at T2
4. Adiabatic expansion back to T1

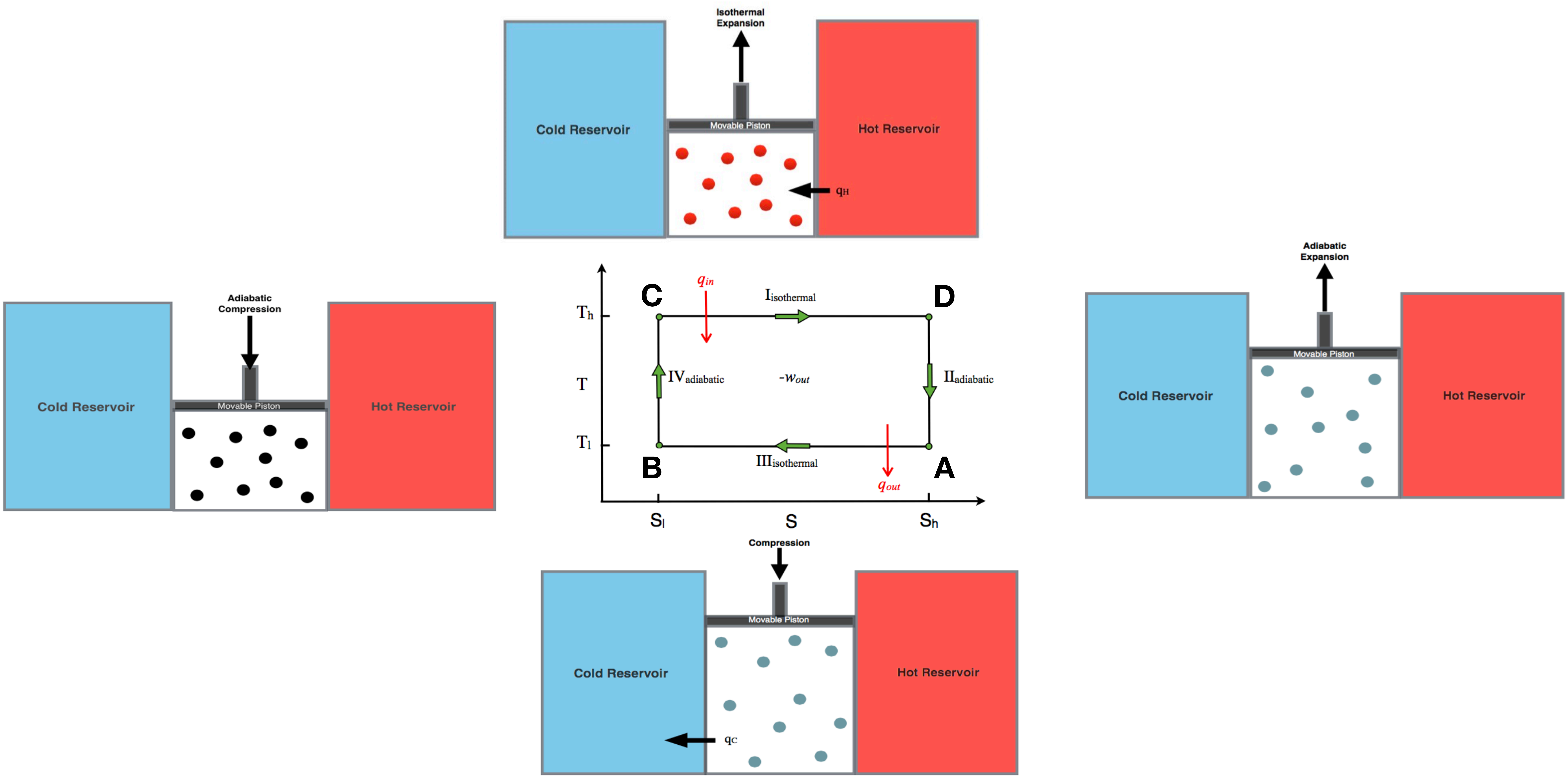


Carnot Engine



1. Isothermal compression at a cooler T1
2. Adiabatic compression to T2
3. Isothermal expansion at T2
4. Adiabatic expansion back to T1

Carnot Engine



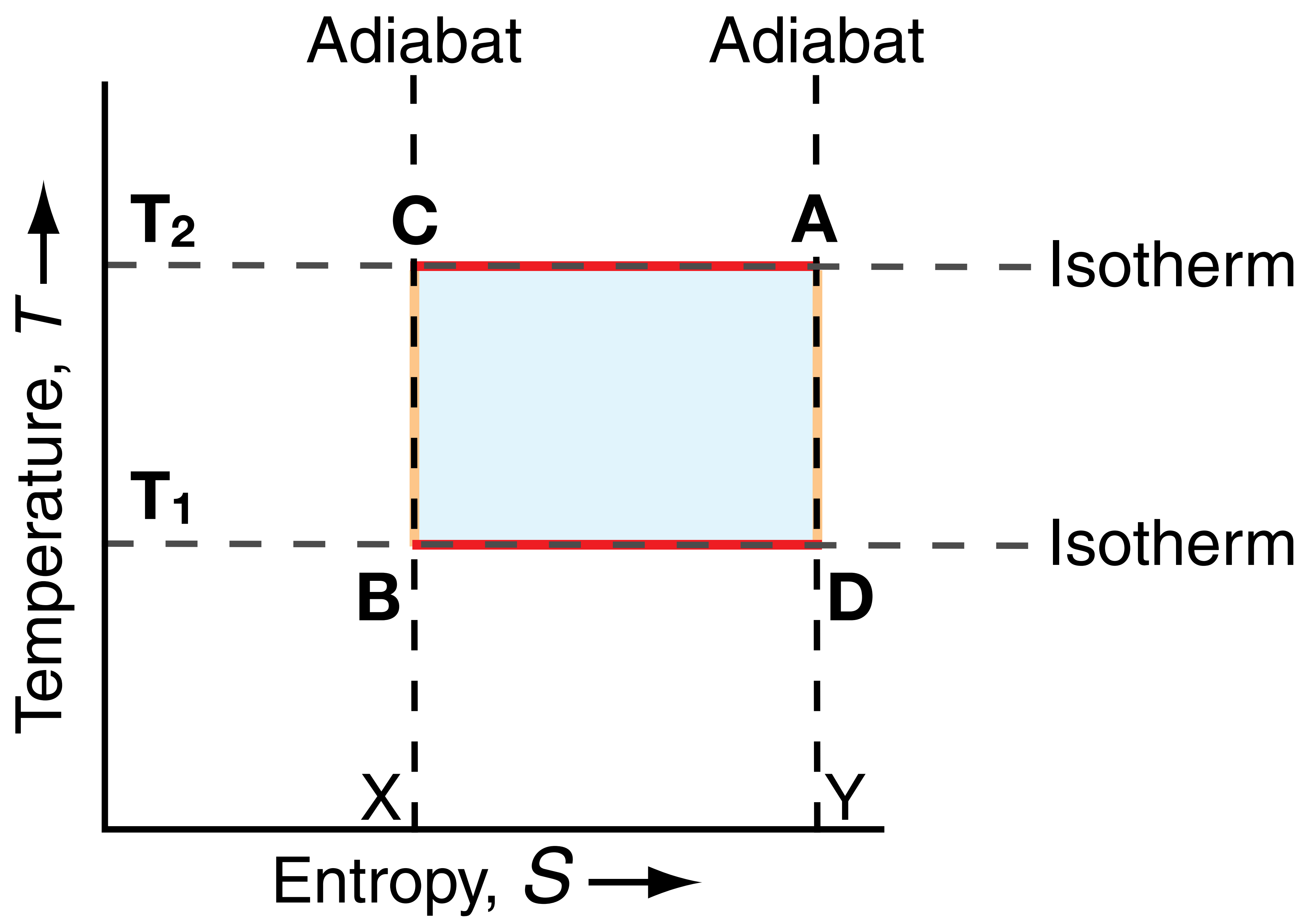
Carnot Engine

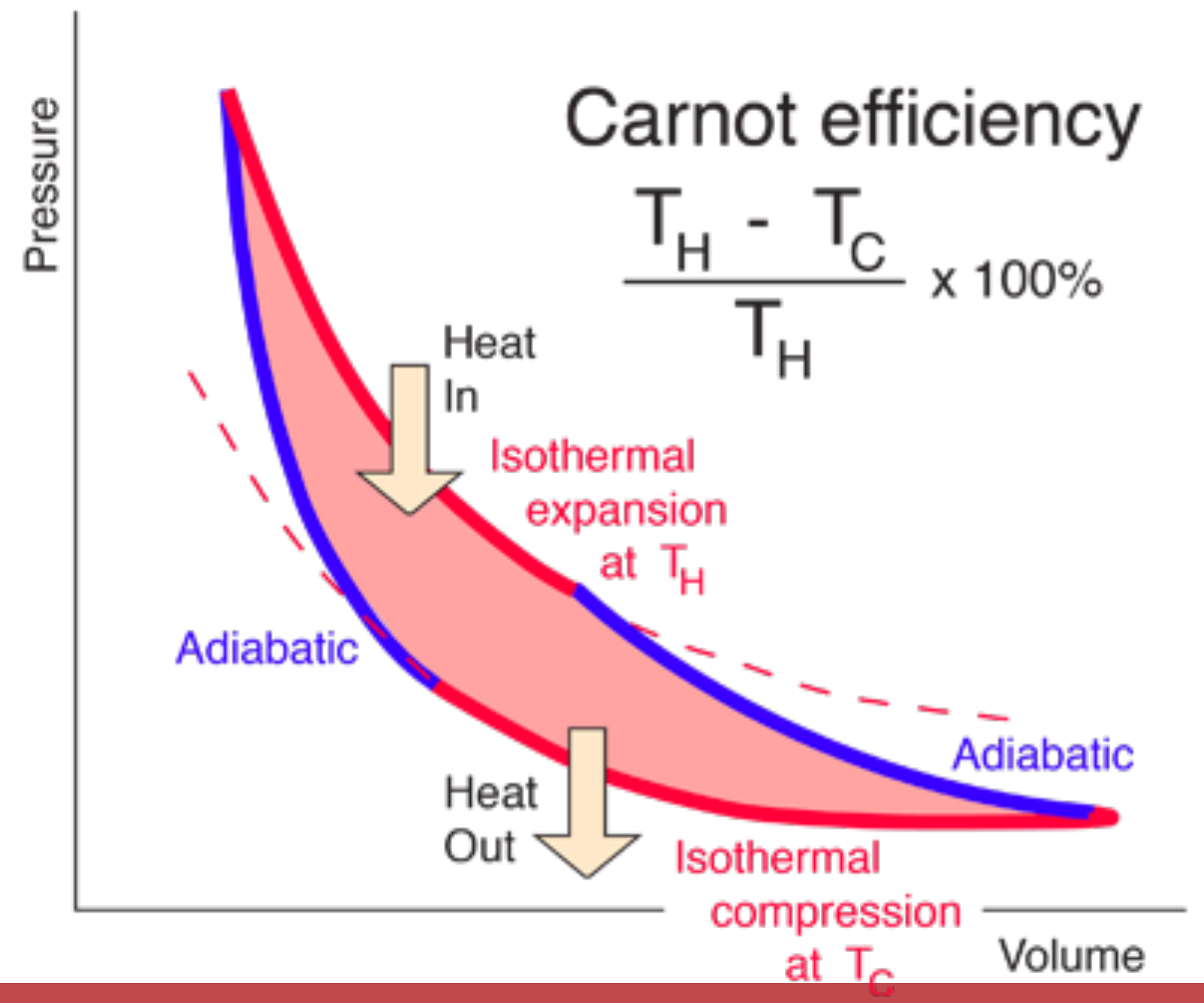
By expanding the integral into the four components of the cycle we find that

$$W = \oint p d\alpha = q_{in} - q_{out} = \varepsilon T_1 (s_{in} - s_{out})$$

$$\varepsilon = \frac{\textbf{Work done}}{\textbf{Heat absorbed}} = \frac{T_1 - T_2}{T_1}$$

Is the Carnot Efficiency





Carnot Engine

All the processes in the Carnot engine are reversible.

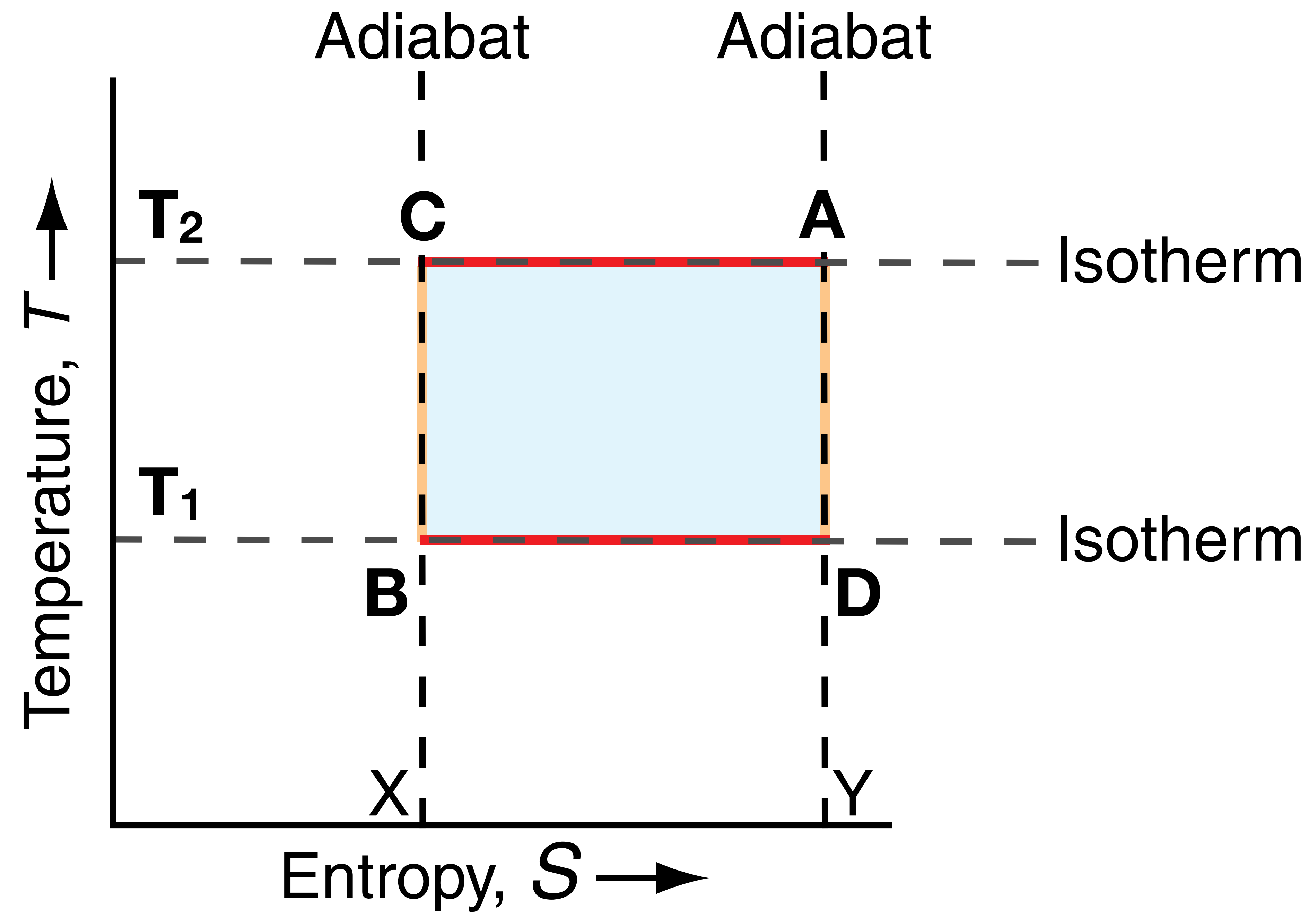
In reality, processes are irreversible, which causes additional heat loss.

Thus

$$\varepsilon = \frac{\textbf{Work done}}{\textbf{Heat absorbed}} = \frac{T_1 - T_2}{T_1}$$

Is the maximum efficiency that is possible in a given system.

$$\varepsilon(\text{Reality}) < \varepsilon(\text{Carnot})$$



We will now begin discussing the last topic of the course:

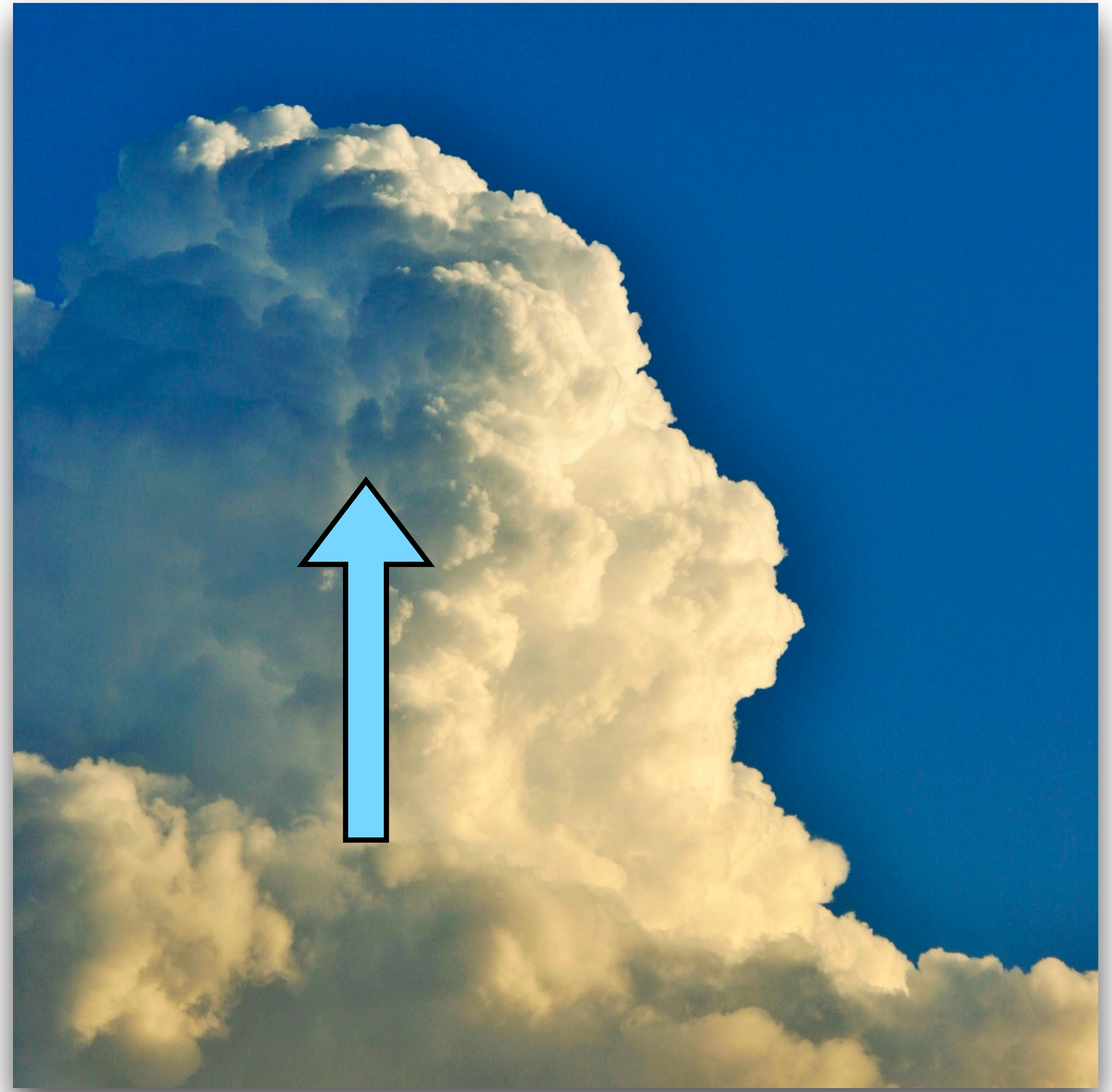
buoyancy and convection

Vertical Acceleration

Newton's second law dictates that acceleration must result from a net sum of forces.

Apply this to vertical motion

$$\frac{Dw}{Dt} = \frac{1}{m}\Sigma_i F_z$$

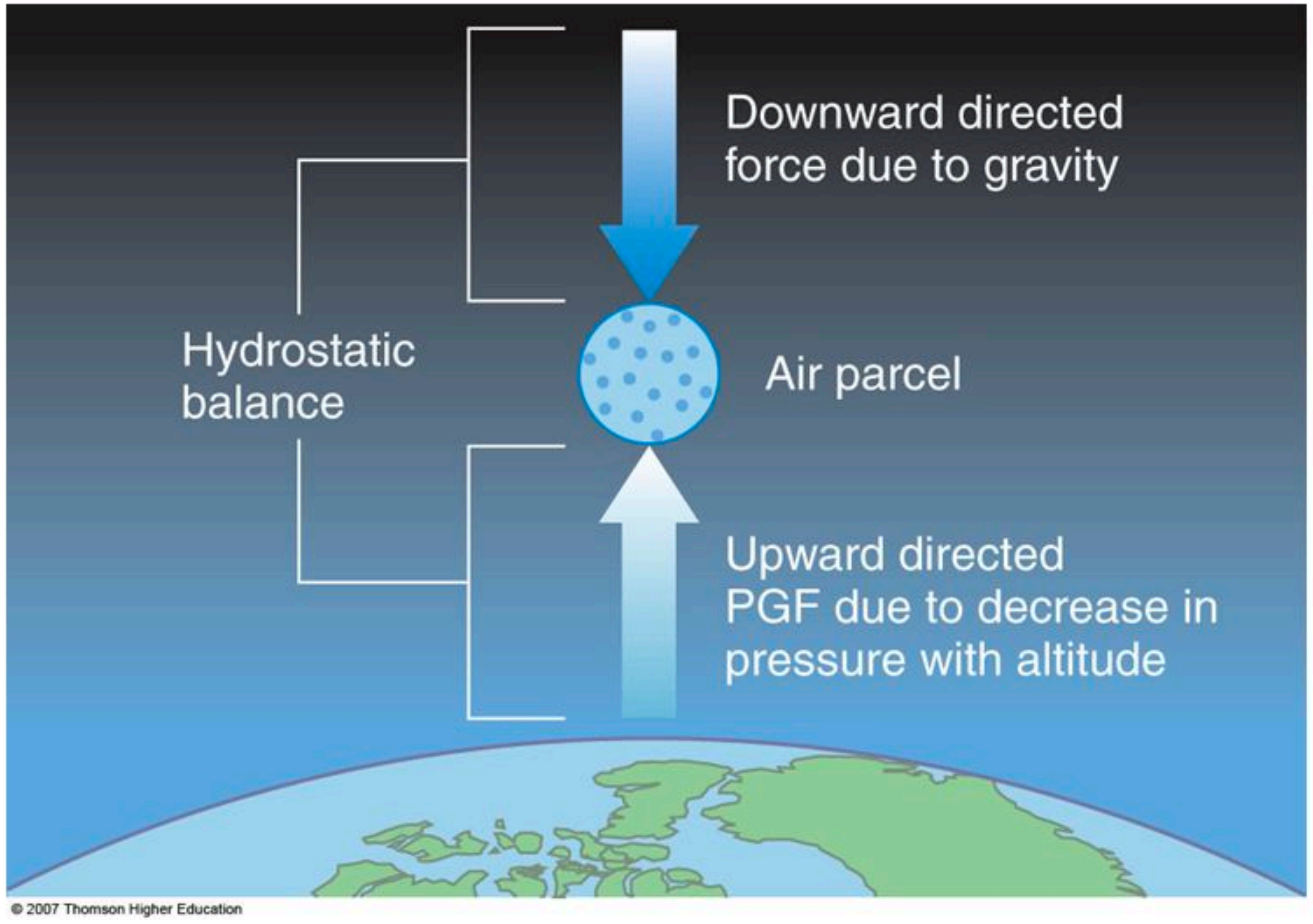


Vertical Acceleration

Ignoring the effects of planetary rotation and friction, the two main forces that cause vertical acceleration are gravity and the pressure gradient force.

$$\frac{Dw}{Dt} = -\alpha\frac{\partial p}{\partial z} - g$$

Acceleration | Pressure gradient force | Gravity



Hydrostatic Balance

For quiescent atmospheric conditions, the atmosphere is maintained in place by a balance between the **downward gravitational force** and the **upward pressure gradient force.**

$$\rho g \simeq -\frac{\partial p}{\partial z}$$

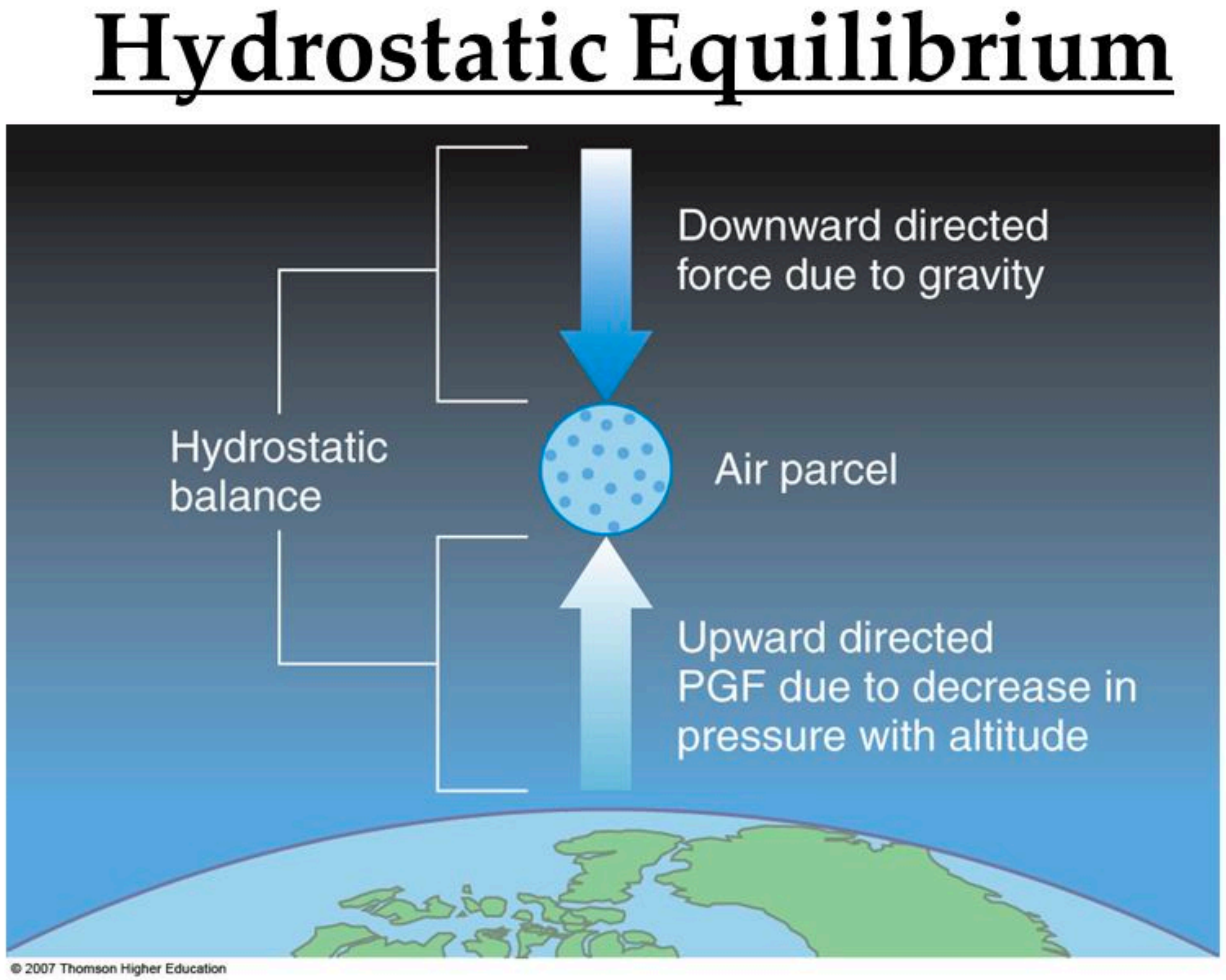


Fig. 6-13, p. 171

Defining buoyancy

We want to know how a parcel can accelerate upward. **If we assume a hydrostatic atmosphere** as the atmosphere's mean state, then

$$\rho_0 g \equiv -\frac{\partial p_0}{\partial z}$$

$$\rho = \rho_0 + \rho'$$

$$p = p_0 + p'$$

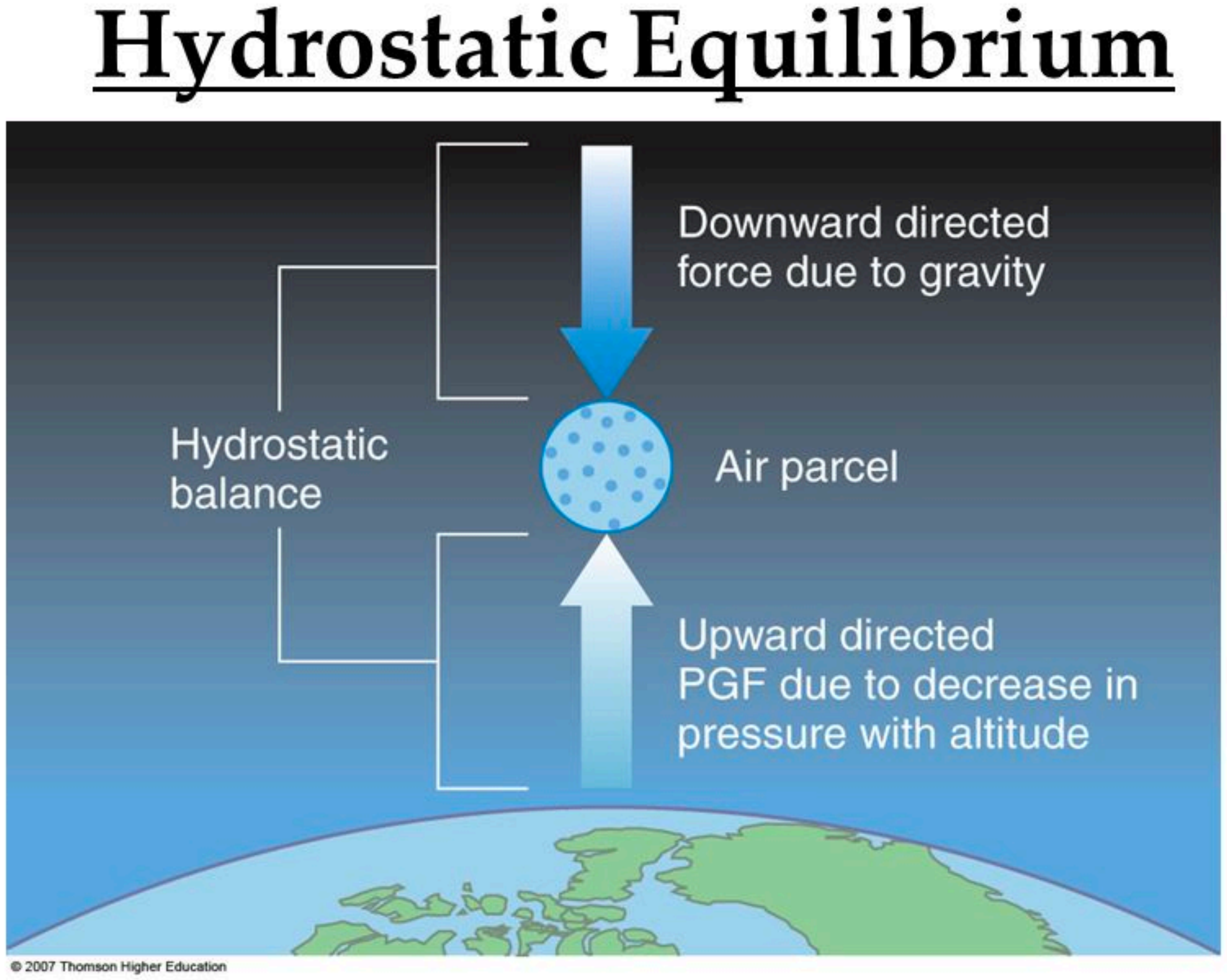


Fig. 6-13, p. 171

Defining buoyancy

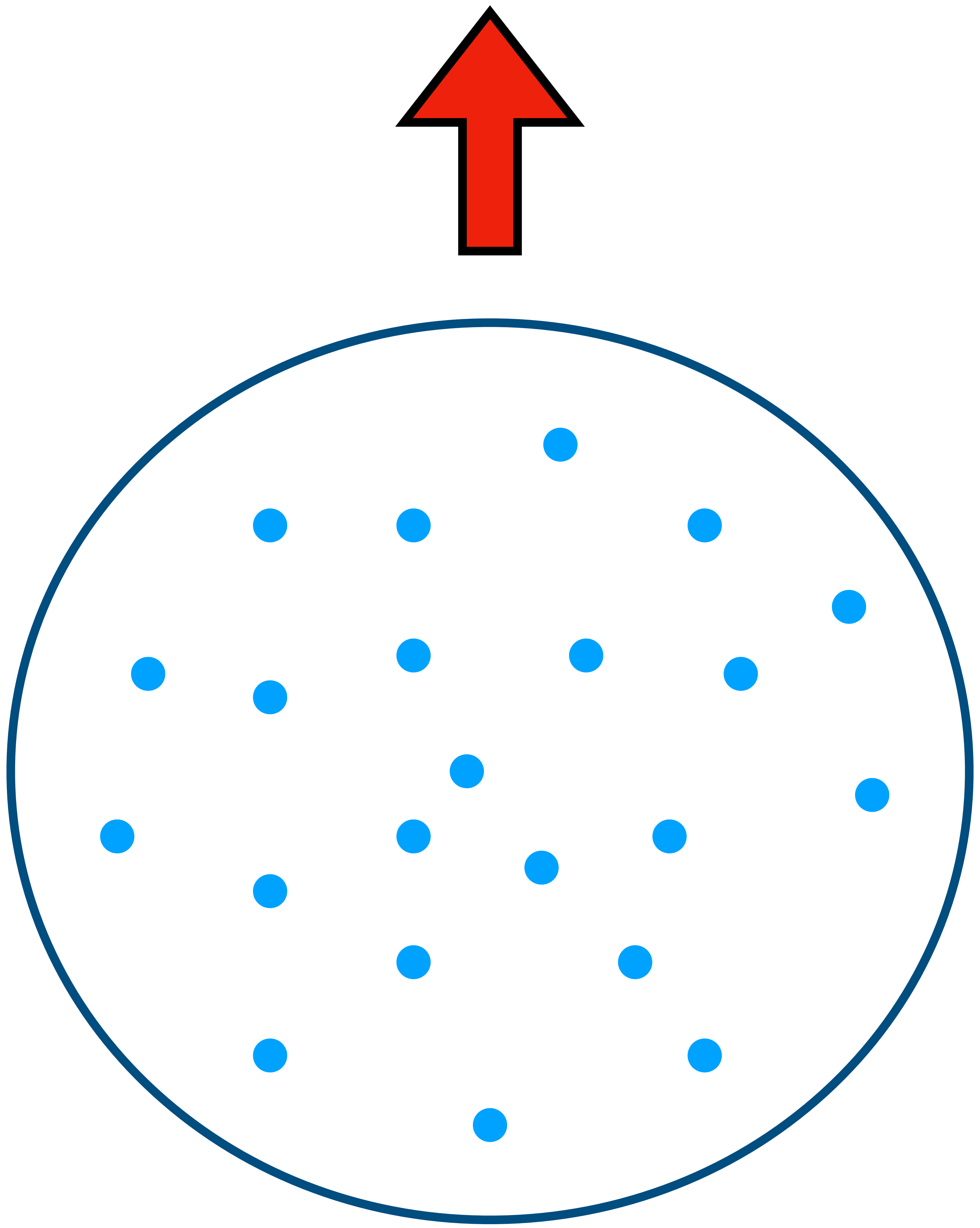
$$\frac{Dw}{Dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + B$$

Perturbation Pressure gradient force

Buoyancy

$$B \equiv -g\frac{\rho - \rho_0}{\rho_0}$$

How can a parcel accelerate upward?



Note: The book uses F_B for buoyancy

For moist unsaturated air

Can express the buoyancy as the difference in virtual temperature between the parcel and its surroundings.

$$B \simeq g \frac{T_v - T_{v0}}{T_{v0}}$$