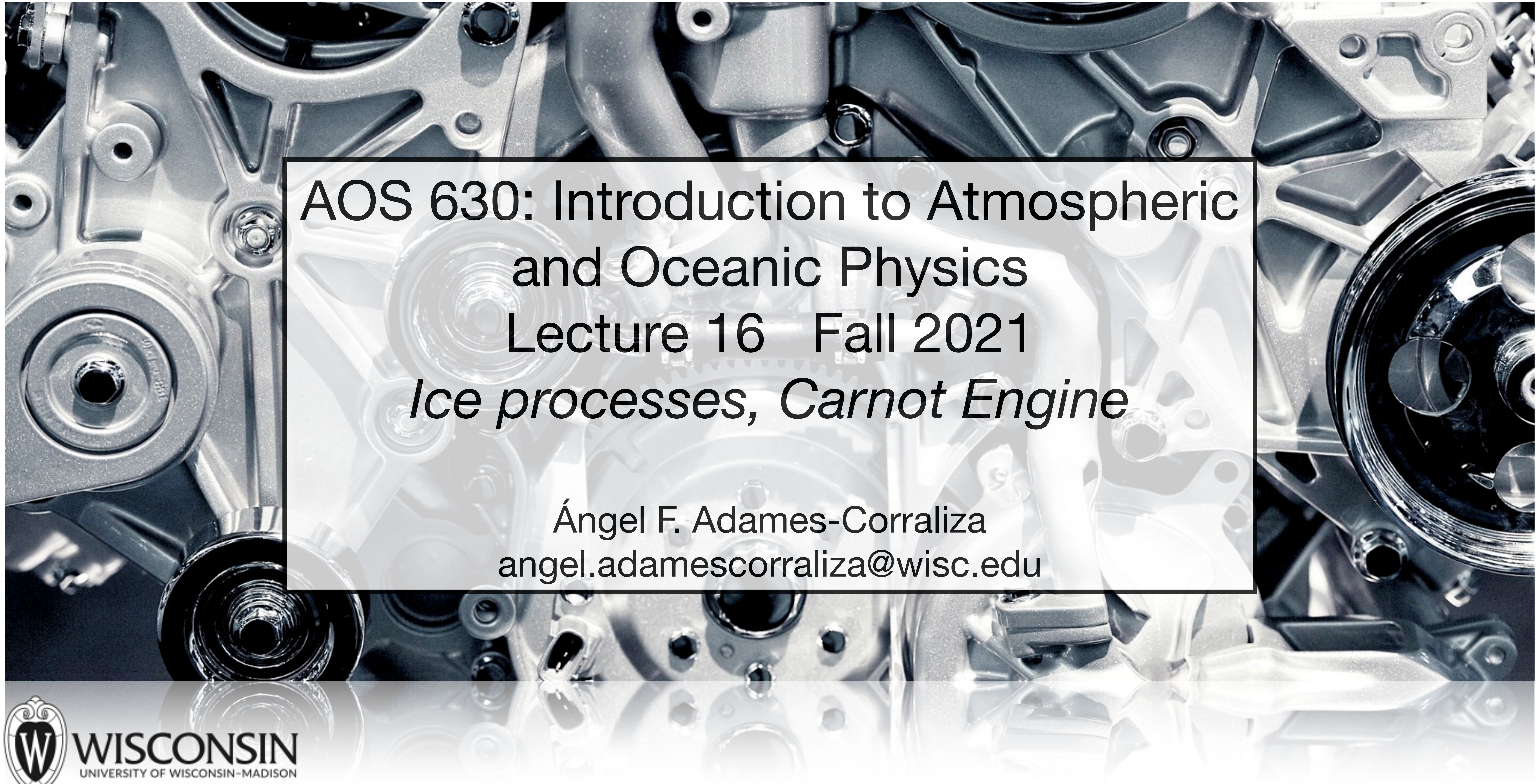

AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 16   Fall 2021

Ice processes, Carnot Engine

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Announcements

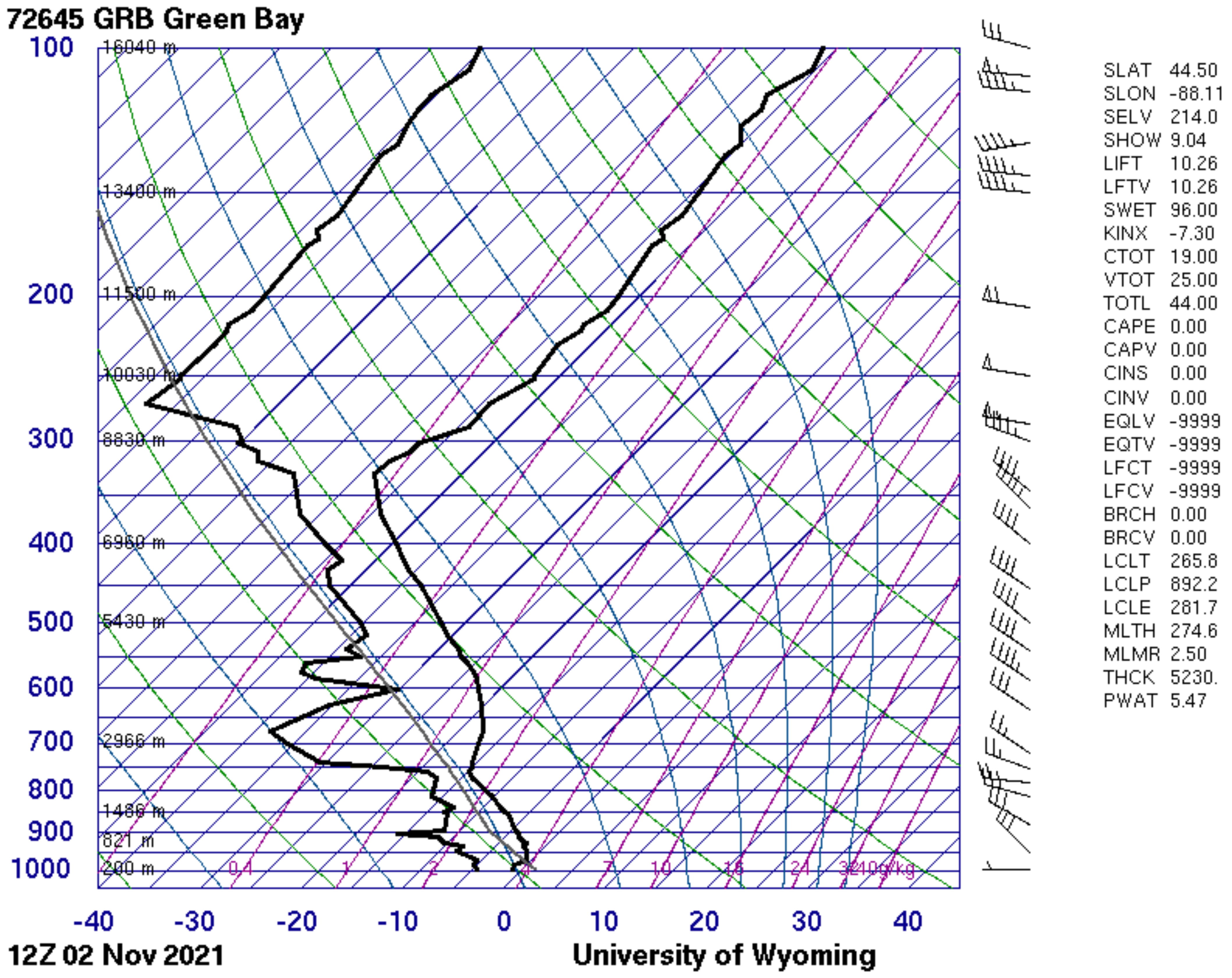
Skew-T a week is due today. Please submit by the end of the day.

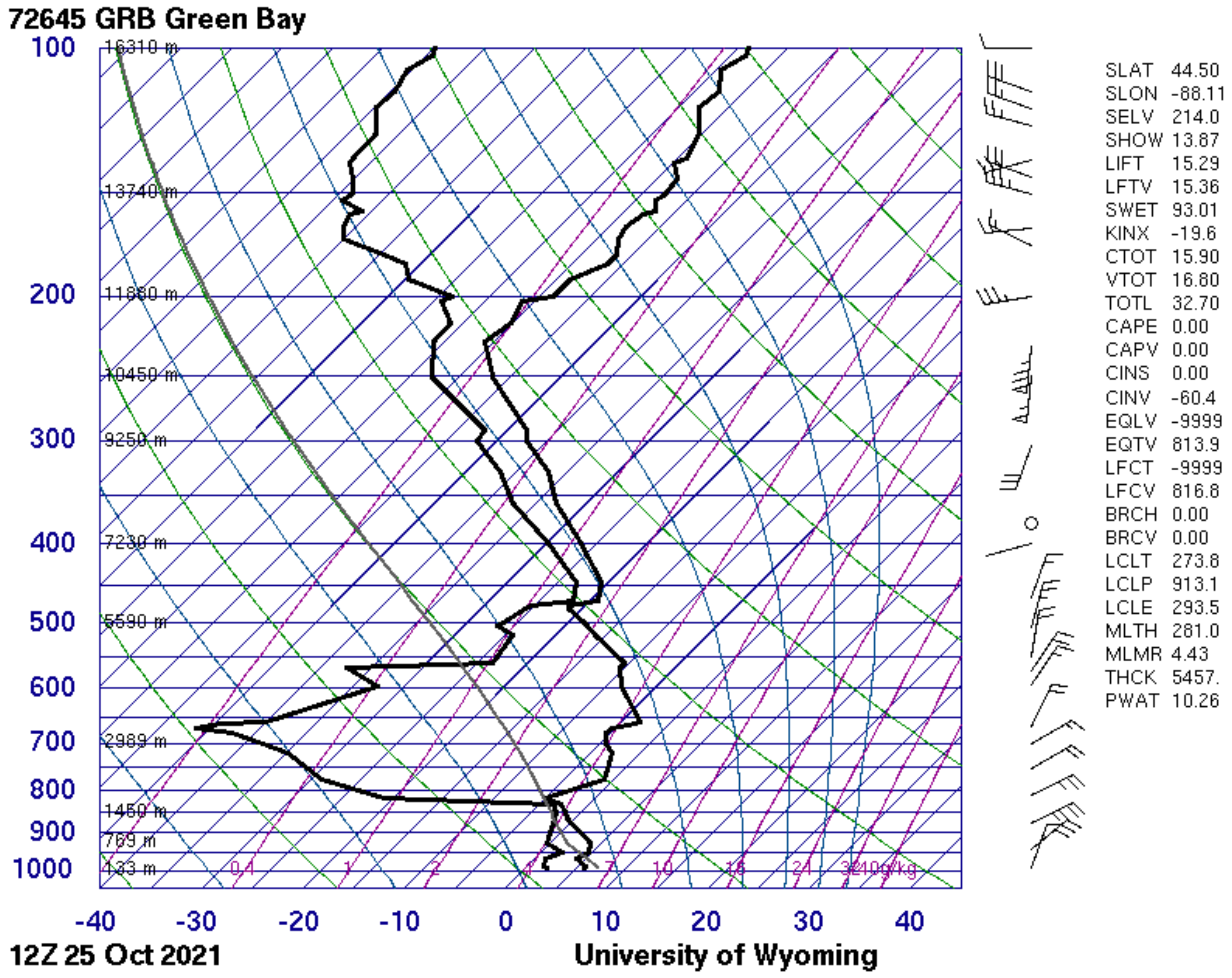
Next Tuesday we will discuss the section titled “**The mature hurricane: A natural Carnot engine**” by Emanuel (1991) (TC_Carnot_Engine.pdf on Canvas).

Please come prepared.

Please meet with me after class (office hours) if you have anything you want to discuss about the final.

Today’s Green Bay sounding vs October 25. What do you see?





Last class

$$c_p \frac{DT}{Dt} - \alpha \frac{Dp}{Dt} = -L_v \frac{Dq_v}{Dt} + \dot{Q}_e$$

$$\dot{Q}_e = -L_f(m - f + s - d) + \dot{Q}_r + \mathcal{F}_h + \mathcal{F}_q$$

We can get a simpler conserved variable is we assume hydrostatic balance:

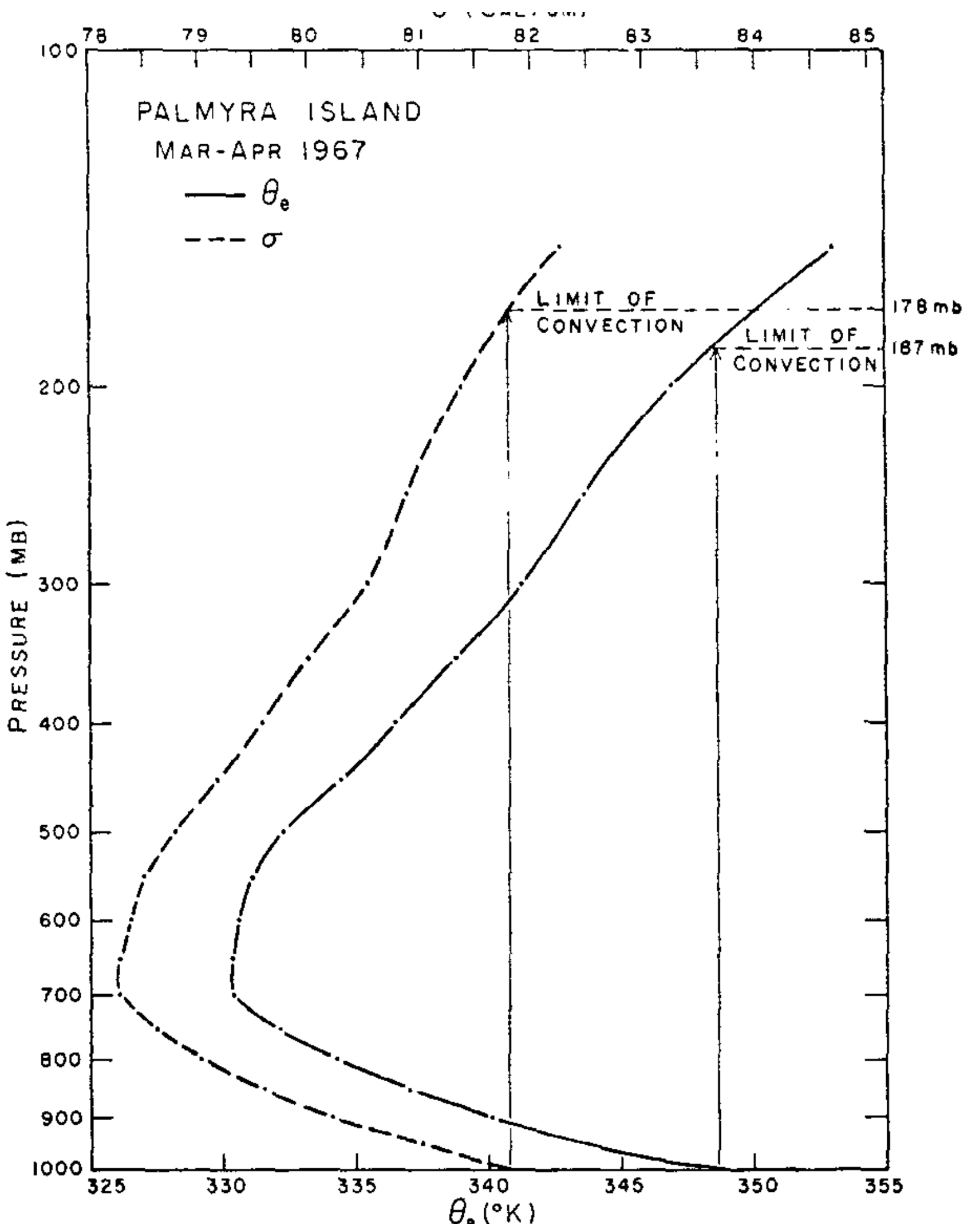
$$\frac{Dp}{Dt} \simeq \frac{dp}{dz}\frac{Dz}{Dt} = -\rho g \frac{Dz}{Dt} = -\rho \frac{D\Phi}{Dt}$$

Which we can plug into the first equation above to obtain

$$\frac{Dm}{Dt} = \dot{Q}_e$$ Where $$m = c_pT + \Phi + L_v q_v$$

The moist static energy

MSE vs theta-e

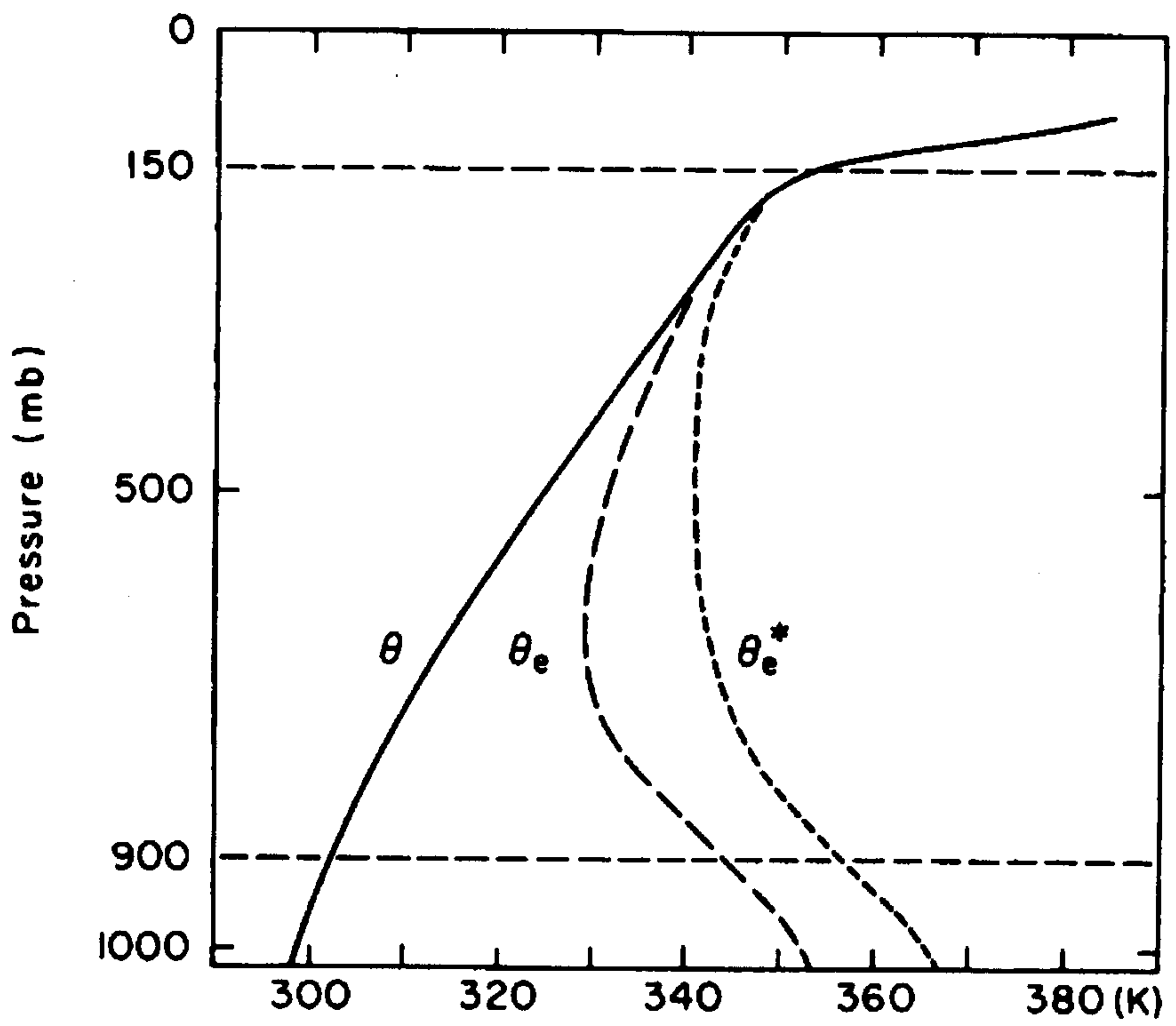


A tropical sounding is shown on the left

The two variables look very similar

The maximum near the surface is due to the large concentration of water vapor.

Profile of theta-e



Water vapor changes the profile of theta-e

$$\theta_e \simeq \theta \exp\left(\frac{L_v q_v}{c_p T}\right)$$

The moist adiabatic lapse rate

A parcel that rises moist adiabatically conserved its MSE as it rises

$$\frac{Dm}{Dz} = 0$$

By expanding the definition and after some algebra and rearranging

$$\frac{dT}{dz} = -\Gamma_m \qquad \Gamma_m = \Gamma_d \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}}$$

Is the **moist adiabatic lapse rate.**

Today

Briefly discuss ice processes

Introduce the Carnot Engine

Moist static energy

Let’s return to the MSE budget (we can later generalize)

$$\frac{Dm}{Dt} = \dot{Q}_e \qquad \dot{Q}_e = -\boxed{L_f(m - f + s - d)} + \dot{Q}_r + \mathscr{F}_h + \mathscr{F}_q$$

There are still processes that are inherent to phase changes within.
Parcel

Can we define a quantity that is conserved even for ice processes?

Invoke water continuity

The total water content is

$$q_T = q_v + q_l + q_i$$

q_T **Specific total water mass content**

q_v **Specific humidity**

q_l **Specific liquid water content**

q_i **Specific ice content**

$$\frac{Dq_t}{Dt} = \frac{Dq_v}{Dt} + \frac{Dq_l}{Dt} + \frac{Dq_i}{Dt} = 0$$

$$\frac{Dq_v}{Dt} = e - c + s - d$$

$$\frac{Dq_l}{Dt} = c - e + m - f$$

$$\frac{Dq_i}{Dt} = f - m + d - s$$

Invoke water continuity

$$\frac{Dm}{Dt} = \dot{Q}_e$$

$$\dot{Q}_e = -\boxed{L_f(m - f + s - d)} + \dot{Q}_r + \mathscr{F}_h + \mathscr{F}_q$$

$$\frac{Dq_t}{Dt} = \frac{Dq_v}{Dt} + \frac{Dq_l}{Dt} + \frac{Dq_i}{Dt} = 0$$

$$\frac{Dq_v}{Dt} = e - c + s - d$$

$$\frac{Dq_l}{Dt} = c - e + m - f$$

$$\frac{Dq_i}{Dt} = \boxed{f - m + d - s}$$

The remaining terms are those that define the ice continuity equation.

Frozen Moist Static Energy

The MSE and ice content budget can be combined to obtain the **frozen MSE** budget

$$\frac{Dm_f}{Dt} = \dot{Q}_r + \mathscr{F}_h + \mathscr{F}_q \qquad m_f = c_p T + \Phi + L_v q_v - L_f q_i$$

It is conserved for all transformations of water. You can obtain a frozen potential temperature and a frozen moist entropy in a similar way.

You will see this quantity frequently used in studies of tropical deep convection.

The importance of ice in deep clouds is a recent realization (last 20 years).

Heat engines:
The Carnot Cycle

Supplemental reading

Petty Section Sec 5.5

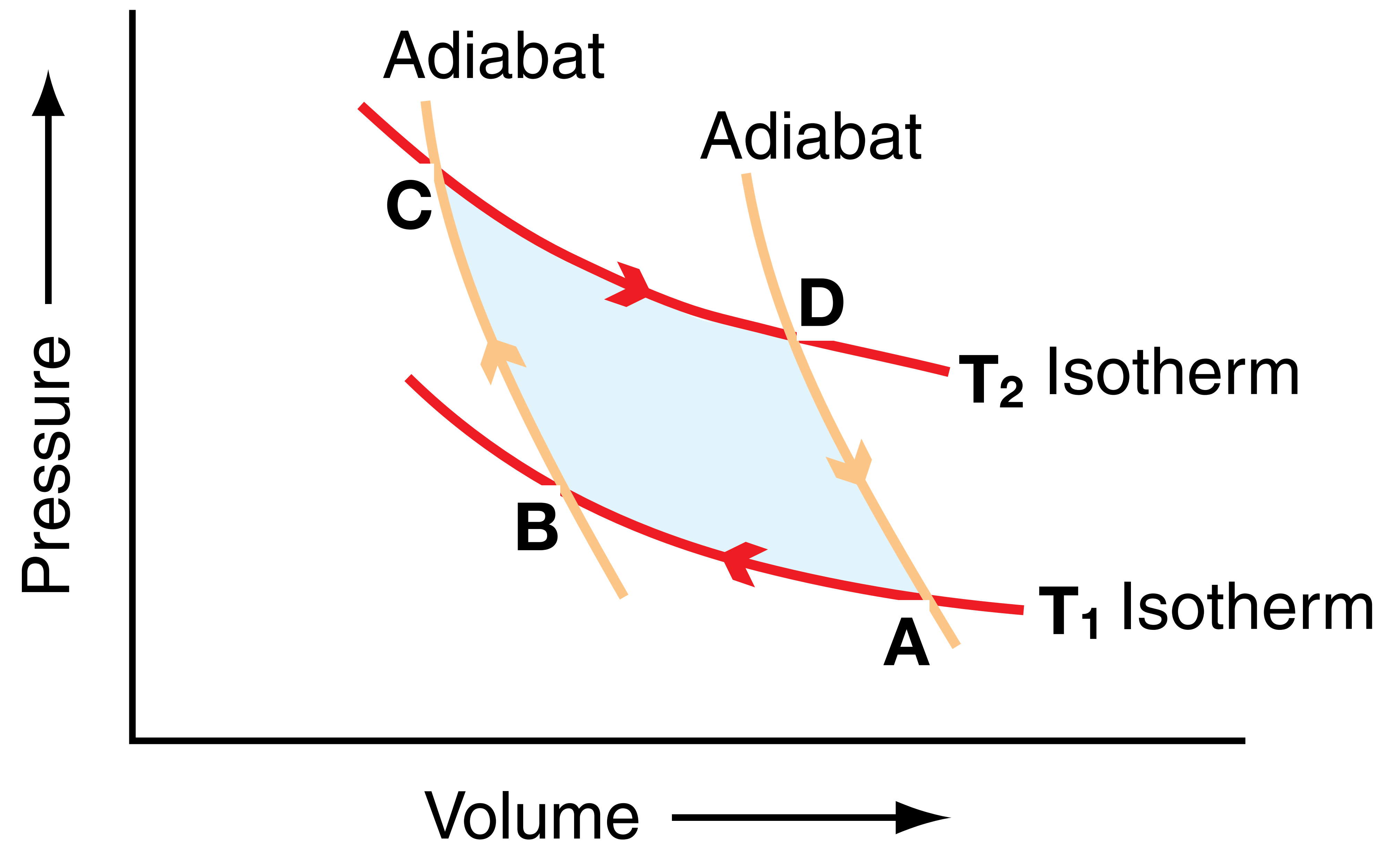
Wallace and Hobbs Sec 3.7

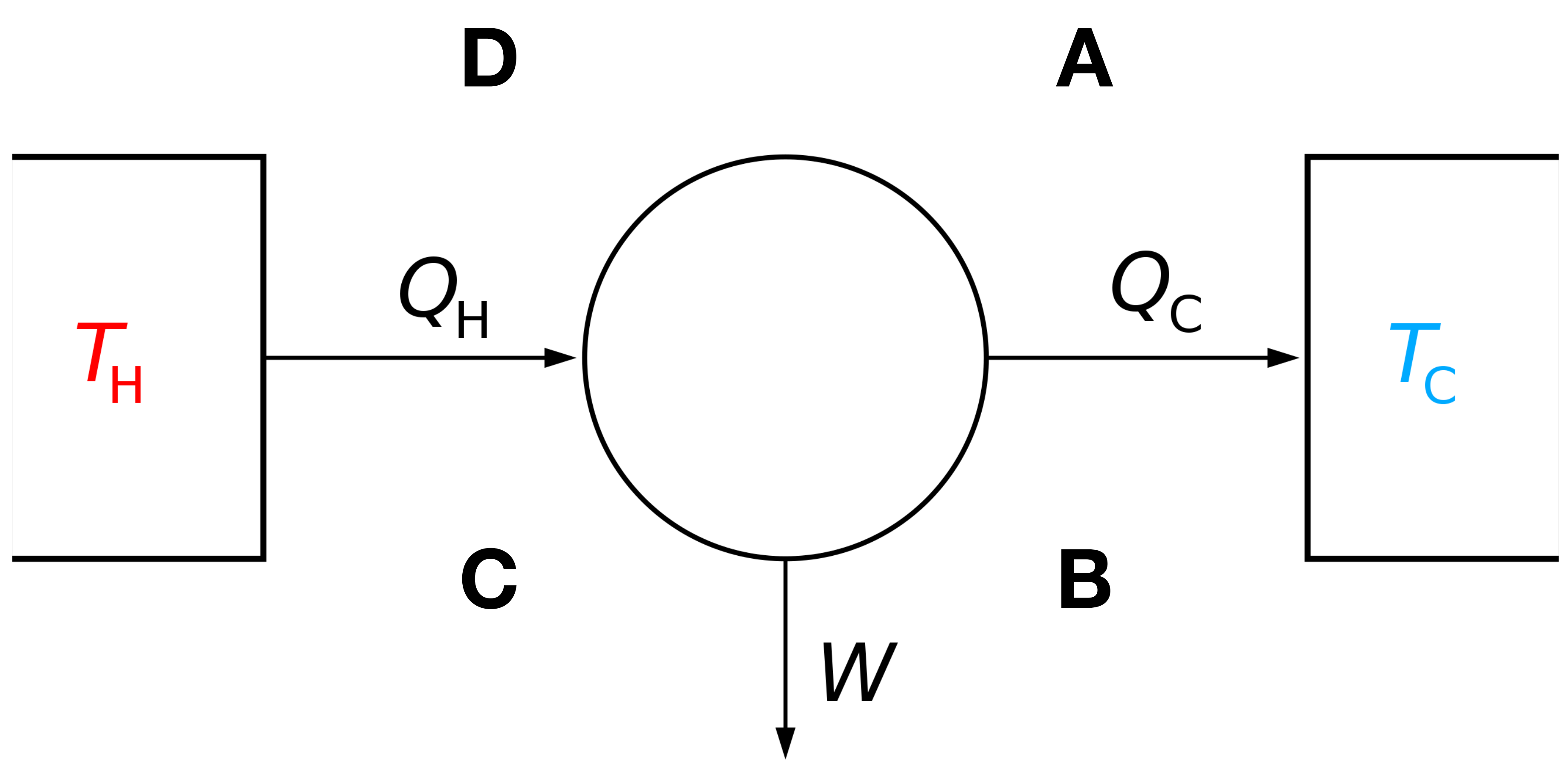
Carnot Engine

Consider a cyclical process involving an ideal gas.

The cycle goes through four steps, as outlined in the diagram on the right.

Essentially, you input heat at a high temperature and the system does work in proportion to the amount of input heat.



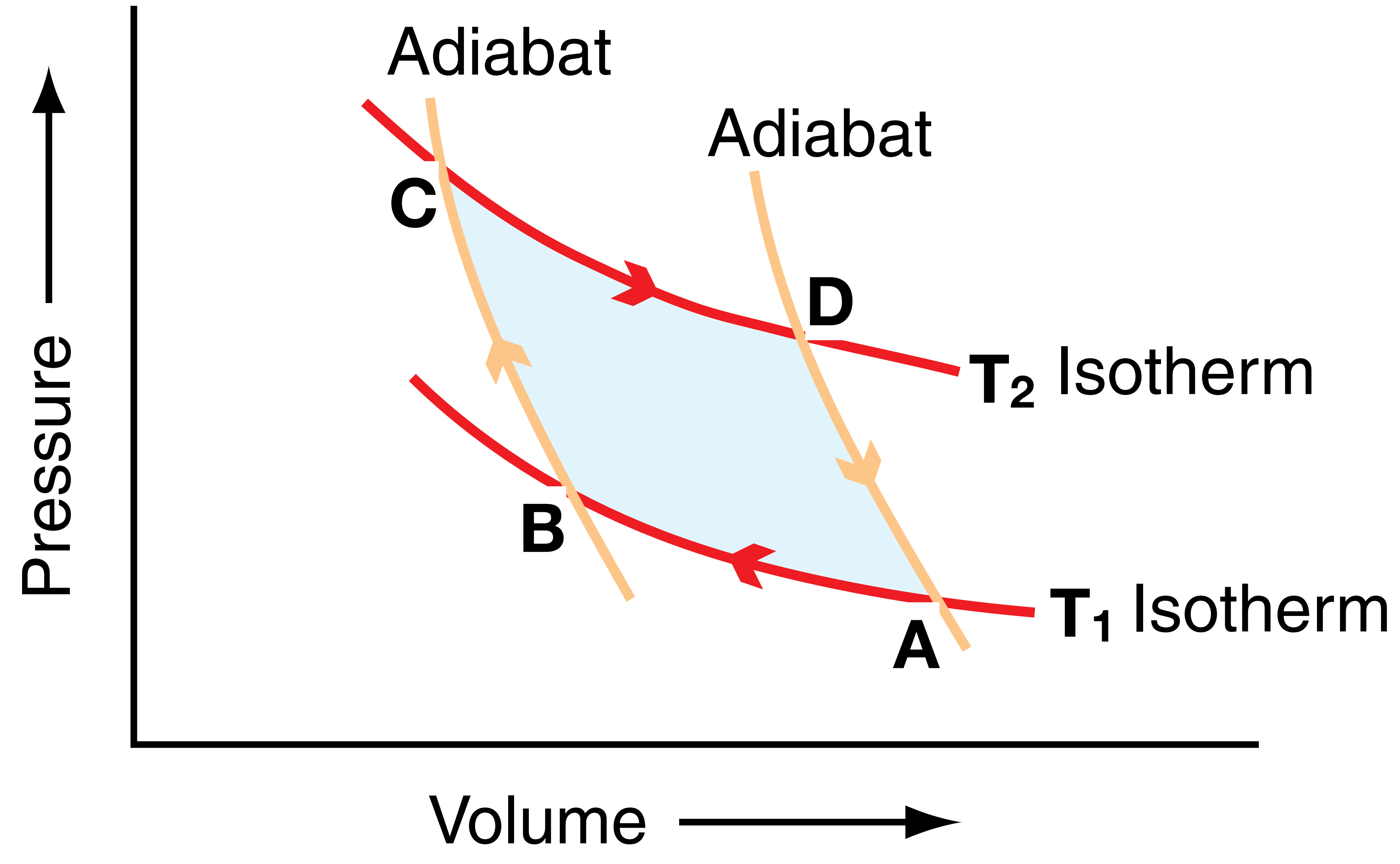


The first law (internal energy form) integrated over this cycle takes the form:

$$\oint c_v dT = \oint \delta q - \oint \delta w$$

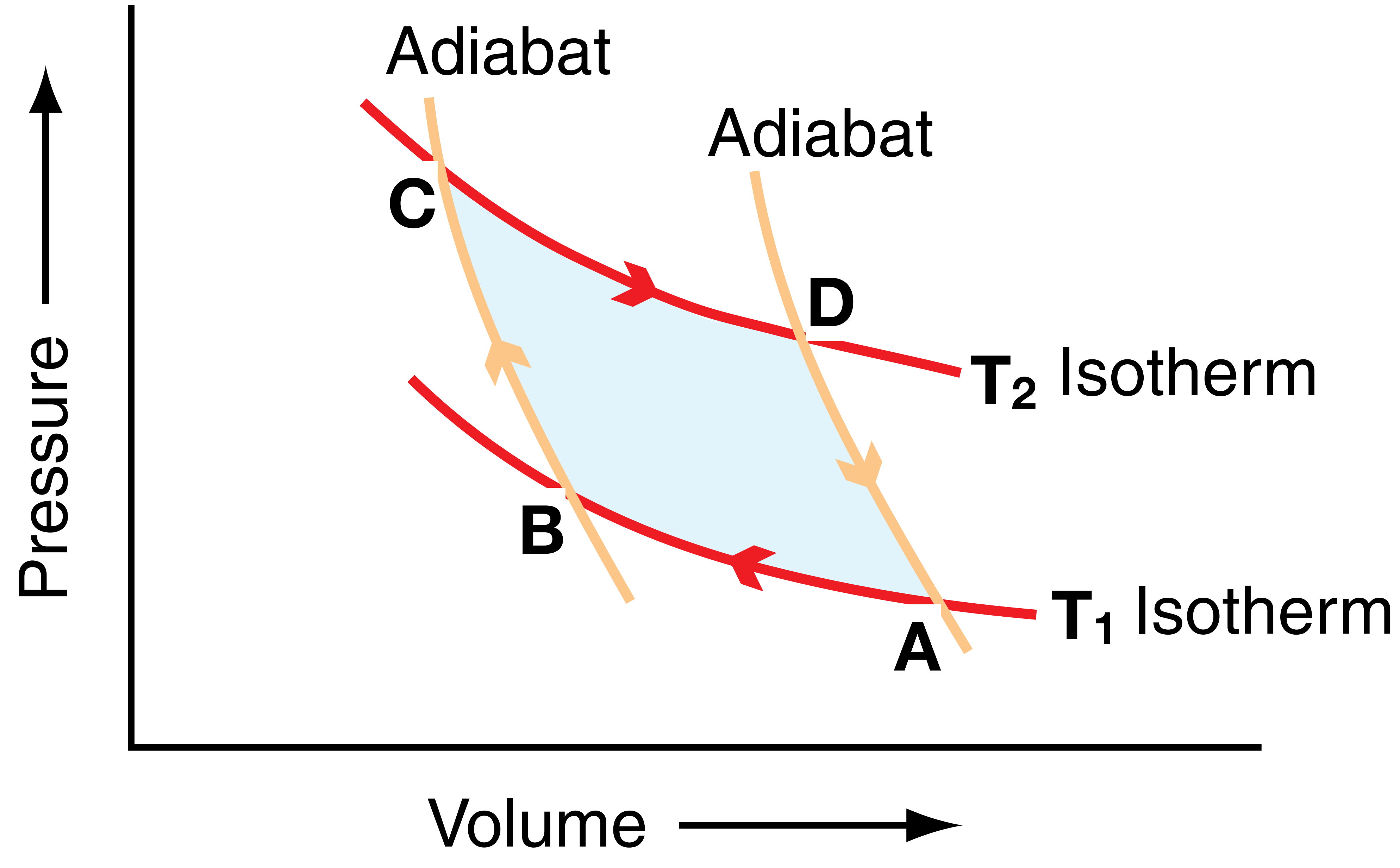
Because state variable don't change during a closed loop integral, it follows that

$$\oint \delta q = \oint \delta w$$



Because the variables in the integral are process variables, it follows that

$$\oint \delta q = \oint \delta w \neq 0$$



The cycle can do work in proportion to the amount of heat input.

In HW4 you get to think about the above premise but for a hurricane. What is the heat input and what is the work in that case?