The role of ice in adiabatic processes

Let's begin by considering the MSE budget:

$$\frac{Dm}{Dt} = \dot{Q}_e \qquad m = MSE = c_pT + \Phi + L_v q_v$$

$$\dot{Q}_e = L_f(f - m + d - s) + \dot{Q}_r + \mathcal{F}_g + \mathcal{F}_h$$

what happens when we consider these "ice processes"

We can define a variable like MSE that is also conserved for "ice processes".

Define the specific ice content as $q_i = \frac{M_i}{M_{air}}$

The ice budget: $\frac{Dq_i}{Dt} = S_i = f - m + d - s$

We can now expand the MSE budget:

$$\frac{Dm}{Dt} = L_f\frac{Dq_i}{Dt} + \dot{Q}_r + \mathcal{F}_g + \mathcal{F}_h$$

$$\frac{D}{Dt}(m - L_f q_i) = \dot{Q}_r + \mathcal{F}_g + \mathcal{F}_h$$

We can define a new variable: the frozen moist static energy

$m_F = FMSE = m - L_f q_i$ conserved under ice processes and moist adiabatic processes.

$= c_pT + \Phi + L_v q_v - L_f q_i$

It follows that $L_v q_v > L_f q_i$ under most conditions

However, $L_f q_i$ is not negligible

$L_v q_v \rightarrow$ the energy that you obtain if you condense all water vapor

$L_f q_i \rightarrow$ the energy that you lose if you melt your ice content

The Carnot Cycle:

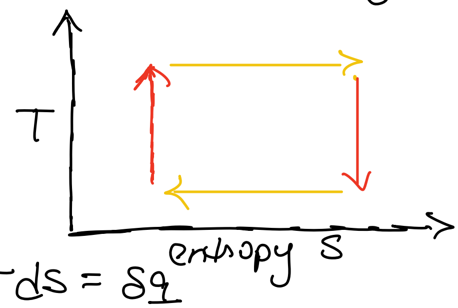
Consider a cyclical process. We can write this as:

$$\oint c_v dT = \oint \delta q - \oint \delta w$$ The "internal energy" form of the 1st law

Because we are doing a cycle, $c_v T$ does not change.
It follows that:

$$\oint \delta q = \oint \delta w$$

heat input = work done



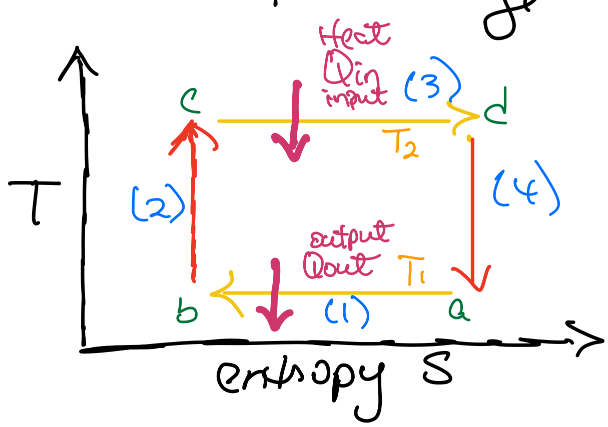
Remember that $ds = \frac{\delta q}{T} \longrightarrow T ds = \delta q$

$$\oint T ds = \oint \delta w$$

heat input = work done

We can divide our cycle into four parts:

1) Isothermal compression at a cooler T_1
2) Adiabatic compression from T_1 to T_2
3) Isothermal expansion at a warmer T_2
4) Adiabatic expansion back to T_1



$$\oint T ds = \oint p\, d\alpha$$ we can break the integral into its four steps

$$= \int_a^b T ds + \int_b^c T ds + \int_c^d T ds + \int_d^a T ds$$

Step (1) $\int_a^b T ds = T_1 \int_a^b ds = T_1 (S_b - S_a) = T_1 \Delta S$

Step (2) $\int_b^c T\,dS = 0$

Step (3) $\int_c^d T\,dS = T_2 \int_c^d dS = T_2 (S_d - S_c)$

Step (4) $\int_d^a T\,dS = 0$