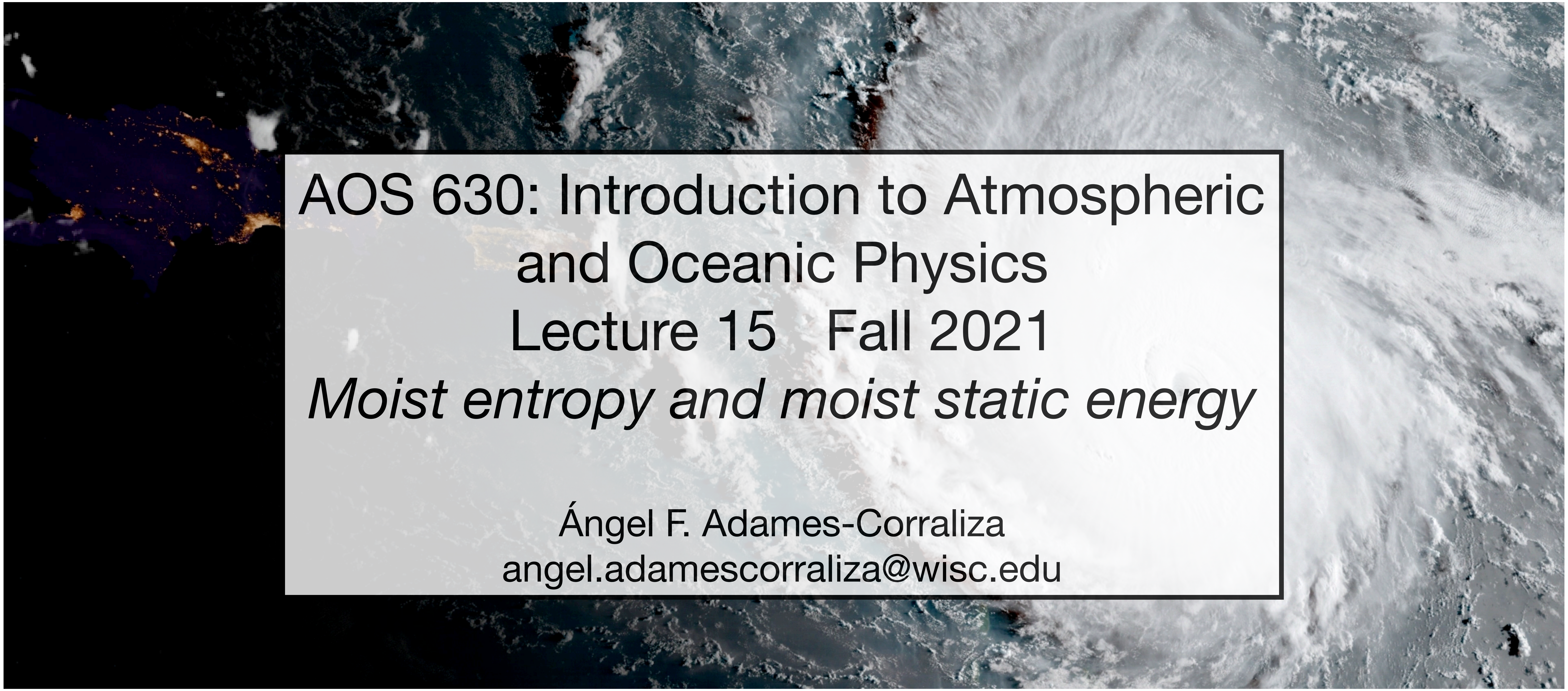


AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 15   Fall 2021

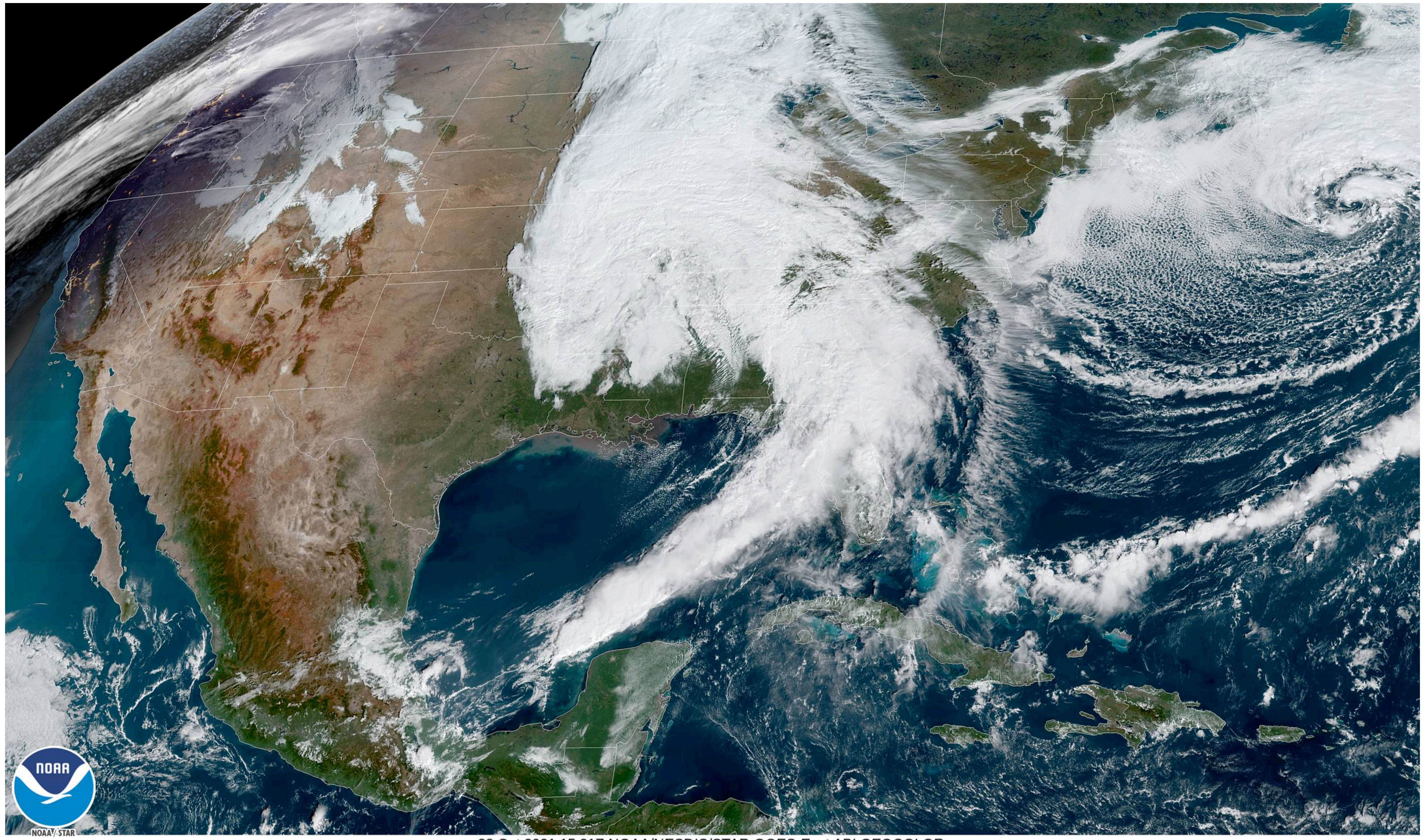
Moist entropy and moist static energy

Ángel F. Adames-Corraliza
angel.adamescorraliza@wisc.edu

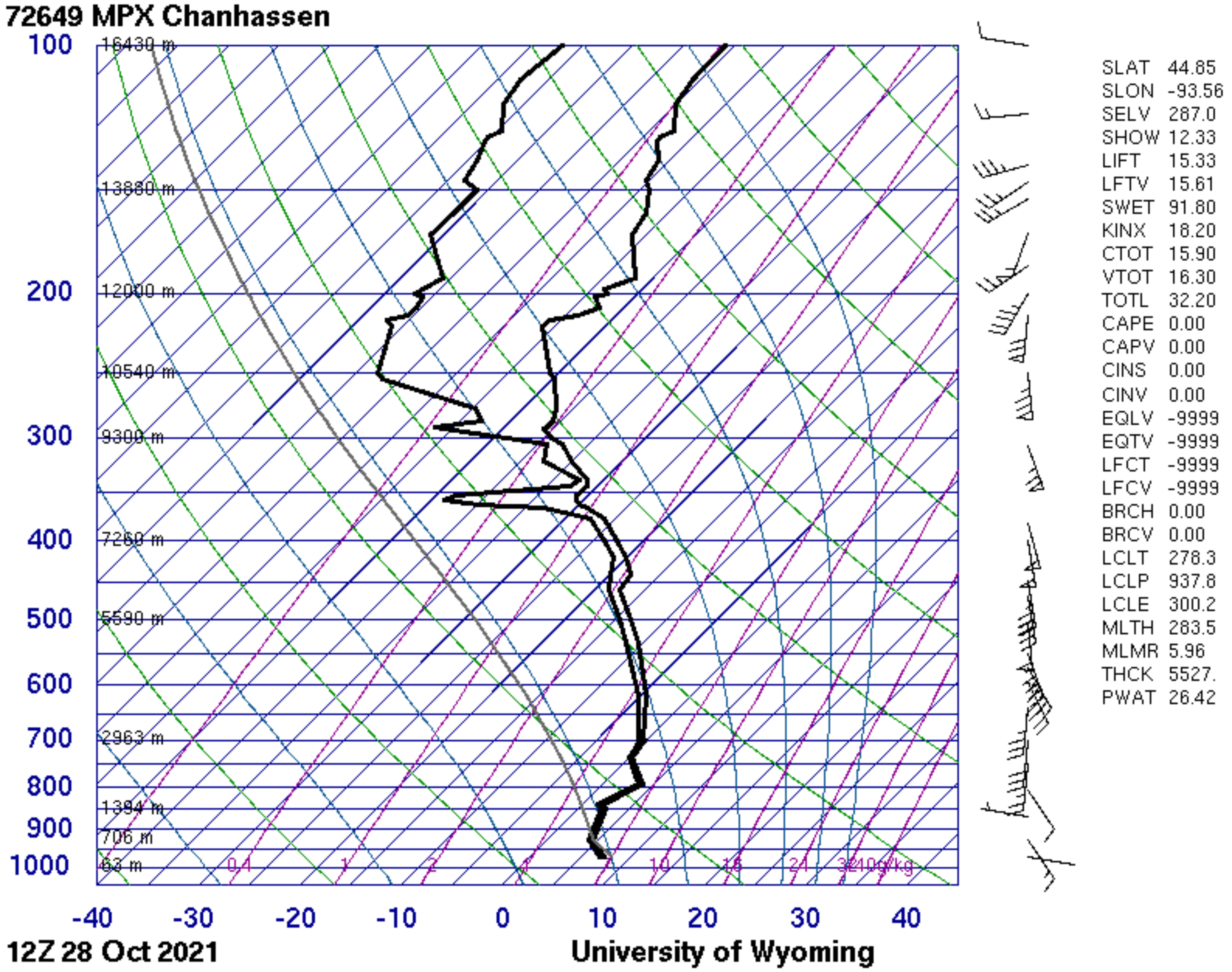
WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Daily dose of thermo

Humid day anyone?



28 Oct 2021 15:21Z NOAA/NESDIS/STAR GOES-East ABI GEOCOLOR



Announcements

HW3 is due today. Please submit by the end of the day.

Please let me know if you can't submit by midnight tonight so I can make accommodations. Otherwise you will get locked out from submitting by midnight.

HW4 has been uploaded. It is due November 18.

Final presentation schedule has been released.

Last Class: The equivalent potential temperature

Need to go back to 1st law to account for condensation

$$c_p dT = -L_v dq_v + \alpha dp$$

Can be written as

$$c_p T d\ln\theta = -L_v dq_v$$

Which can be solved to obtain

$$\theta_e \simeq \theta \exp\left(\frac{L_v q_v}{c_p T}\right)$$

$$\frac{Dq_v}{Dt} = S_q$$

$$S_q = e - c$$

The equivalent potential temperature

The temperature a parcel would have if it condensed all its water vapor and was brought to the surface adiabatically

The equivalent potential temperature

While θ is not conserved once a cloud begins to develop, θ_e **is** conserved.

In fact, θ_e is conserved even if no condensation is occurring. Thus, it is a very useful variable when trying to understand our atmosphere

Moist Entropy

In an earlier lecture we defined the entropy in terms of the potential temperature

$$ds = \frac{\delta q}{T} = c_p d \ln \theta$$

Entropy is **not** directly proportional to θ_e, but **we can define a new entropy that is** using the following form of the first law

$$c_p T d \ln \theta + L_v dq_v = 0$$

Dividing the equation by T yields and rearranging the terms yields the following

$$ds_m = d\left(c_p \ln \theta + \frac{L_v q_v}{T}\right) \qquad s_m = c_p \ln \theta + \frac{L_v q_v}{T} + \text{const}$$

The specific moist entropy

The moist entropy budget

The previous derivation was done assuming that

$$\delta q \simeq L_v(c - e) \qquad\qquad dq_v \simeq e - c$$

In practice this is not true. Let us return to the equations in time derivative form to re-examine them:

$$c_p \frac{DT}{Dt} = \dot{Q} + \alpha \frac{Dp}{Dt} \qquad\qquad \frac{Dq_v}{Dt} = S_q$$

In general

$$\dot{Q} = -L_v(e - c) - L_s(s - d) - L_f(m - f) + \dot{Q}_r + \mathscr{F}_h \qquad\qquad S_q = e - c + s - d + \mathscr{F}_q / L_v$$

$\mathscr{F}_h$ Turbulent enthalpy flux

$\dot{Q}_r$ Radiative heating rate

e = evaporation
c = condensation
s = sublimation
d = deposition
m = melting
f = freezing

$\mathscr{F}_q$ Turbulent flux of q_v

The moist entropy budget

Some terms are are equal and opposite in the thermodynamic and moisture equations

$$\dot{Q} = -L_v(e-c) - L_s(s-d) - L_f(m-f) + \dot{Q}_r + \mathscr{F}_h \qquad S_q = e - c + s - d + \mathscr{F}_q/L$$

So we can still replace them in the first law

$$c_p\frac{DT}{Dt} - \alpha\frac{Dp}{Dt} = -L_v\frac{Dq_v}{Dt} - L_f(m - f + s - d) + \dot{Q}_r + \mathscr{F}_h + \mathscr{F}_q$$

By rearranging the terms and following the derivation of moist entropy from before we get

$$\frac{Ds_m}{Dt} = \frac{\dot{Q}_e}{T}$$

Where $\dot{Q}_e = -L_f(m - f + s - d) + \dot{Q}_r + \mathscr{F}_h + \mathscr{F}_q$

The moist entropy budget

Moist entropy budget: infinitesimal form

$$\frac{Ds_m}{Dt} = \frac{\dot{Q}_e}{T} \longrightarrow ds_m = \frac{\delta q_e}{T}$$

The moist entropy is a state variable

Recall that state variables follow the integral rule

$$\oint ds_m = 0$$

The moist static energy budget

$$c_p \frac{DT}{Dt} - \alpha \frac{Dp}{Dt} = - L_v \frac{Dq_v}{Dt} + \dot{Q}_e$$

We can get a simpler conserved variable is we assume hydrostatic balance:

$$\frac{Dp}{Dt} \simeq \frac{dp}{dz}\frac{Dz}{Dt} = - \rho g \frac{Dz}{Dt} = - \rho \frac{D\Phi}{Dt}$$

Which we can plug into the first equation above to obtain

$$\frac{Dm}{Dt} = \dot{Q}_e$$ Where $$m = c_p T + \Phi + L_v q_v$$

The moist static energy

Relationship between MSE and moist entropy

Given that

$$\frac{Ds_m}{Dt} = \frac{\dot{Q}_e}{T} \quad \text{and} \quad \frac{Dm}{Dt} = \dot{Q}_e$$

It follows that

$$\frac{Ds_m}{Dt} = \frac{1}{T}\frac{Dm}{Dt} \qquad Tds_m = dm$$

In HW4 you will also show that

$$ds_m = c_p d \ln \theta_e$$

The **MSE**, **moist entropy** and θ_e are interrelated

The moist adiabatic lapse rate

A parcel that rises moist adiabatically conserved its MSE as it rises

$$\frac{Dm}{Dz} = 0$$

By expanding the definition and after some algebra and rearranging

$$\frac{dT}{dz} = -\Gamma_m \qquad \Gamma_m = \Gamma_d \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}}$$

Is the **moist adiabatic lapse rate.**

The moist adiabatic lapse rate

We can show that

$$\frac{1 + \dfrac{L_v q_s}{R_d T}}{1 + \dfrac{L_v^2 q_s}{c_p R_v T^2}} < 1$$

Which means that

$$\Gamma_m < \Gamma_d$$

Parcels cool down more slowly when they rise moist adiabatically.