Moist adiabatic processes:

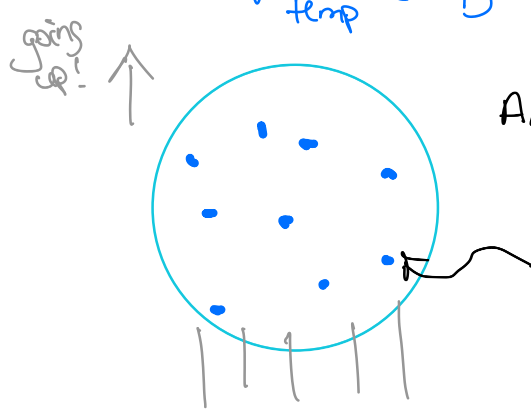
* Consider a parcel that has risen above the LCL. It is saturated, $RH = 100\%$, hence $q_v = q_s$. As the parcel rises, it cools down.
* $q_s(T, P)$, has to follow the Clausius-Clapeyron equation.

$$q_s = \frac{e_s}{P}$$

* The parcel is constrained to cool but to remain at 100% RH (saturated)
* Whatever excess humidity you have as a result of the lifting must condense.

θ, s, s_d are no longer conserved

pot. temp   entropy   Dry static energy



As droplets form, latent energy is released!

Latent heat release

$L_v C$

rate of condensation.

Is there a conservation equation for a parcel that undergoes a moist adiabatic process?

We begin by invoking an equation for the conservation of specific humidity.

$$\frac{Dq_v}{Dt} = S_q$$

S_q = sources and sinks of water vapor.

not the vapor pressure

$$= e - C + S - d + J_q L_v^{-1}$$

evap   cond.   sublimation   deposition

J_q = turbulent flux convergence of $L_v q_v$ exchange of $L_v q_v$ with a body of water or ice.

Let's return to the first law of thermodynamics:
"Enthalpy form" of 1st law in time derivative form:

$$c_p \frac{DT}{Dt} - \alpha \frac{DP}{Dt} = \dot{Q} \qquad (1)$$

Let's consider the case of a parcel that is condensing and evaporating, but otherwise adiabatic.

$\dot{Q} = L_v(c - e)$ all other terms are neglected

$S_q = e - c$ all other terms are neglected

Moisture eqn:

$$\frac{Dq_v}{Dt} = e - c$$

First law:

$$c_p \frac{DT}{Dt} - \alpha \frac{DP}{Dt} = -L_v(e - c)$$

Merging the equations yields:

$$c_p \frac{DT}{Dt} - \alpha \frac{DP}{Dt} = -L_v \frac{Dq_v}{Dt}$$

Use ideal gas law: $p\alpha = R_d T$ (ignore virtual effect)

$$\alpha = \frac{R_d T}{p}$$

$$c_p \frac{DT}{Dt} - \frac{R_d T}{p}\frac{DP}{Dt} = -L_v \frac{Dq_v}{Dt}$$

Move into infinitesimal form:

$$c_p\, dT - \frac{R_d T}{p} dp = -L_v\, dq_v \quad \text{and divide by } T$$

$$c_p \frac{dT}{T} - \frac{R_d}{p} dp = -L_v \frac{dq_v}{T} \quad \text{Div. by } c_p$$

$$\underbrace{d\ln T - \frac{R_d}{c_p} d\ln p}_{= d\ln\theta} = -\frac{L_v}{c_p}\frac{dq_v}{T}$$

$$d\ln\theta = -\frac{L_v}{c_p}\frac{dq_v}{T} \qquad (3)$$

Use the chain rule: $-\frac{L_v}{c_p}\frac{dq_v}{T} = -\frac{L_v}{c_p}\left[d\left(\frac{q_v}{T}\right) + \frac{q_v}{T^2} dT\right]$

Can show that $d\left(\frac{q_v}{T}\right) \gg \frac{q_v}{T^2} dT$ to good accuracy

$\frac{10^{-2}}{10^{2}}$ 10^{-4} $\frac{10^{-2}\ 10^{1}}{10^{5}}$ 10^{-6}

Back to Eq. (3): $d\ln\theta \simeq -d\left(\frac{L_v q_v}{c_p T}\right)$

Integrating $\displaystyle\int_{\theta}^{\theta_e} d\ln\theta \simeq -\int_{q_v}^{0} d\left(\frac{L_v q_v}{c_p T}\right)$

$$\ln\frac{\theta_e}{\theta} = -\left(0 - \frac{L_v q_v}{c_p T}\right)$$

$$\ln\frac{\theta_e}{\theta} = \frac{L_v q_v}{c_p T}$$ Taking exp on both sides:

$$\frac{\theta_e}{\theta} = \exp\left(\frac{L_v q_v}{c_p T}\right)$$

$$\theta_e = \theta\exp\left(\frac{L_v q_v}{c_p T}\right)$$ Equivalent Potential Temperature