AOS 630: Introduction to Atmospheric and Oceanic Physics
Lecture 11   Fall 2021
The Clausius-Clapeyron Equation 2

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Announcements

HW2 is due. Please upload to Canvas by the end of day. Let me know if you need an extension.

HW3 is online. It's a doozy. Start working on it early.

Announcements

Next week we will discuss Held and Soden (2006): **Robust Responses of the Hydrological Cycle to Global Warming.**

Please come ready for a discussion again. It will complement HW3. See Canvas file "Held&Soden2006.pdf"

Supplementary reading

Petty Sections 7.3-7.5

Wallace and Hobbs 3.5.1

Gibbs version of the first law

$$dG = \alpha dp - sdT$$

$$G = h - sT$$ The Gibbs free energy.

Maximum work that can be done by a system at constant temperature and pressure

What advantages might this equation have over other forms of the first law?

Gibbs free energy

We are specifically interested in isobaric, isothermal processes

$$dG = 0$$

Let's think of a system in which contains liquid water and water vapor, and is whose temperature and pressure remain constant.
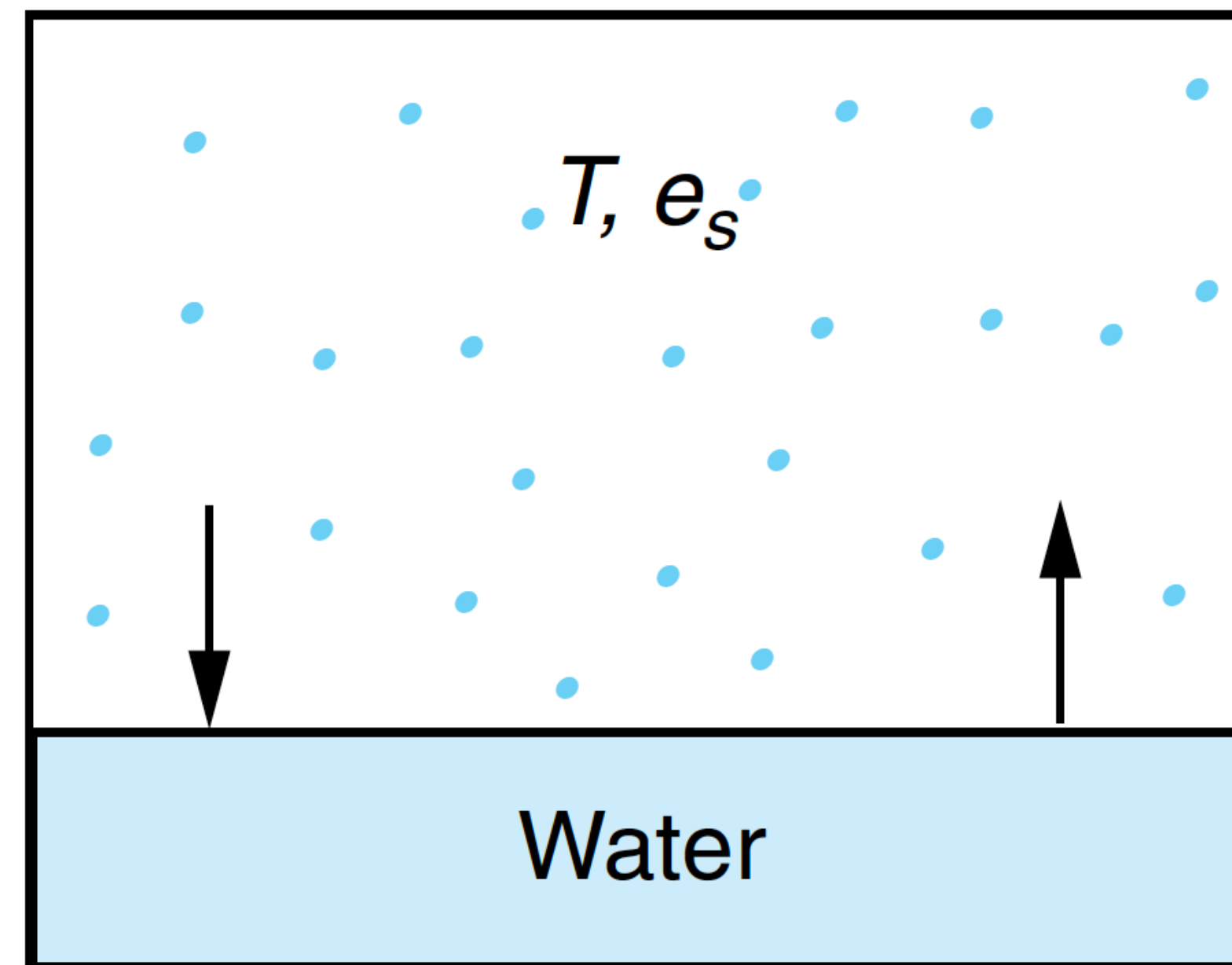
Imagine you have a tank that is completely closed. The tank has liquid water, dry air and water vapor. Evaporation and condensation happen simultaneously in it, but one rate may exceed the other initially.

Eventually the liquid and the gas reach thermodynamic equilibrium.
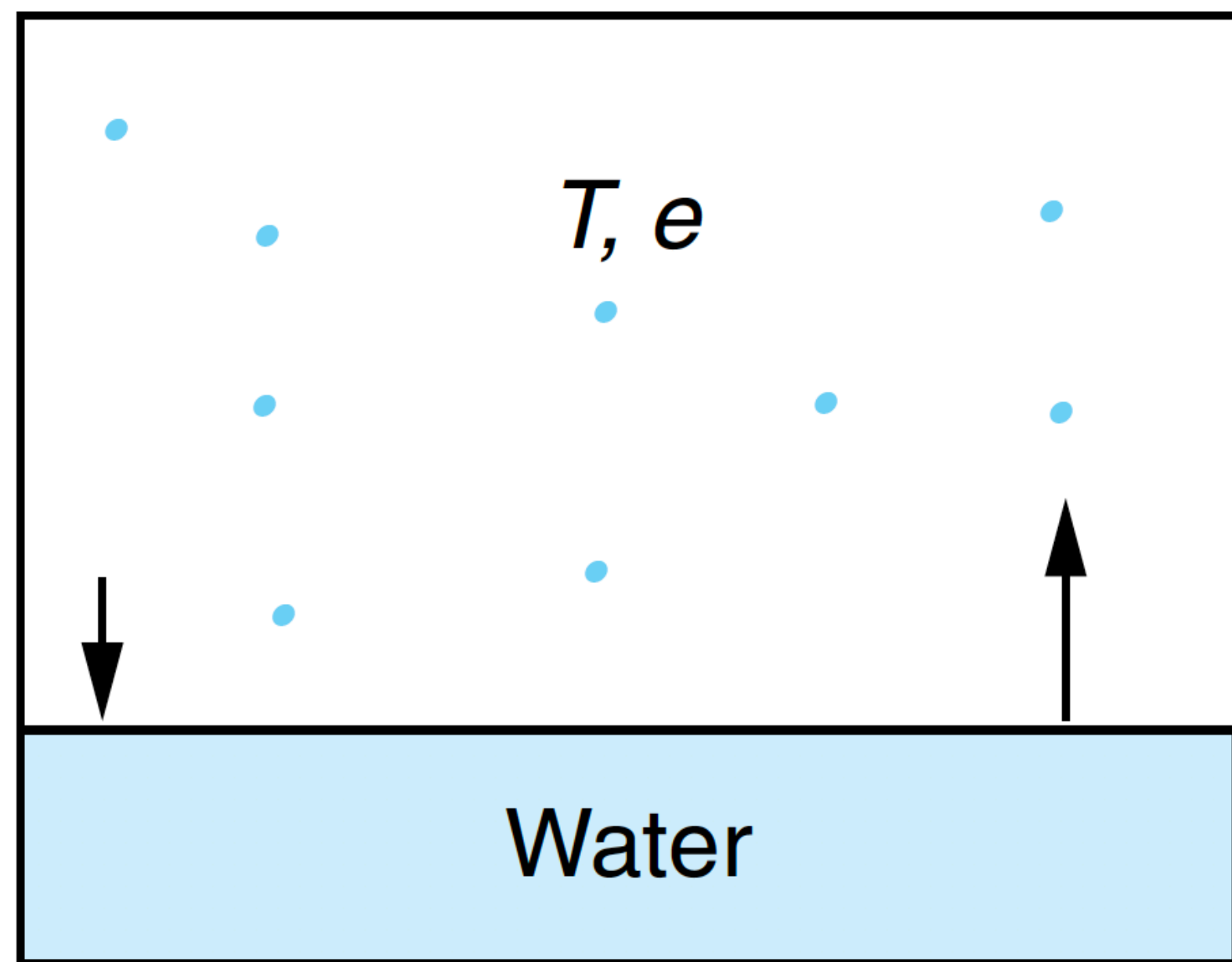
(a) Unsaturated   (b) Saturated   *Wallace and Hobbs (2006)*
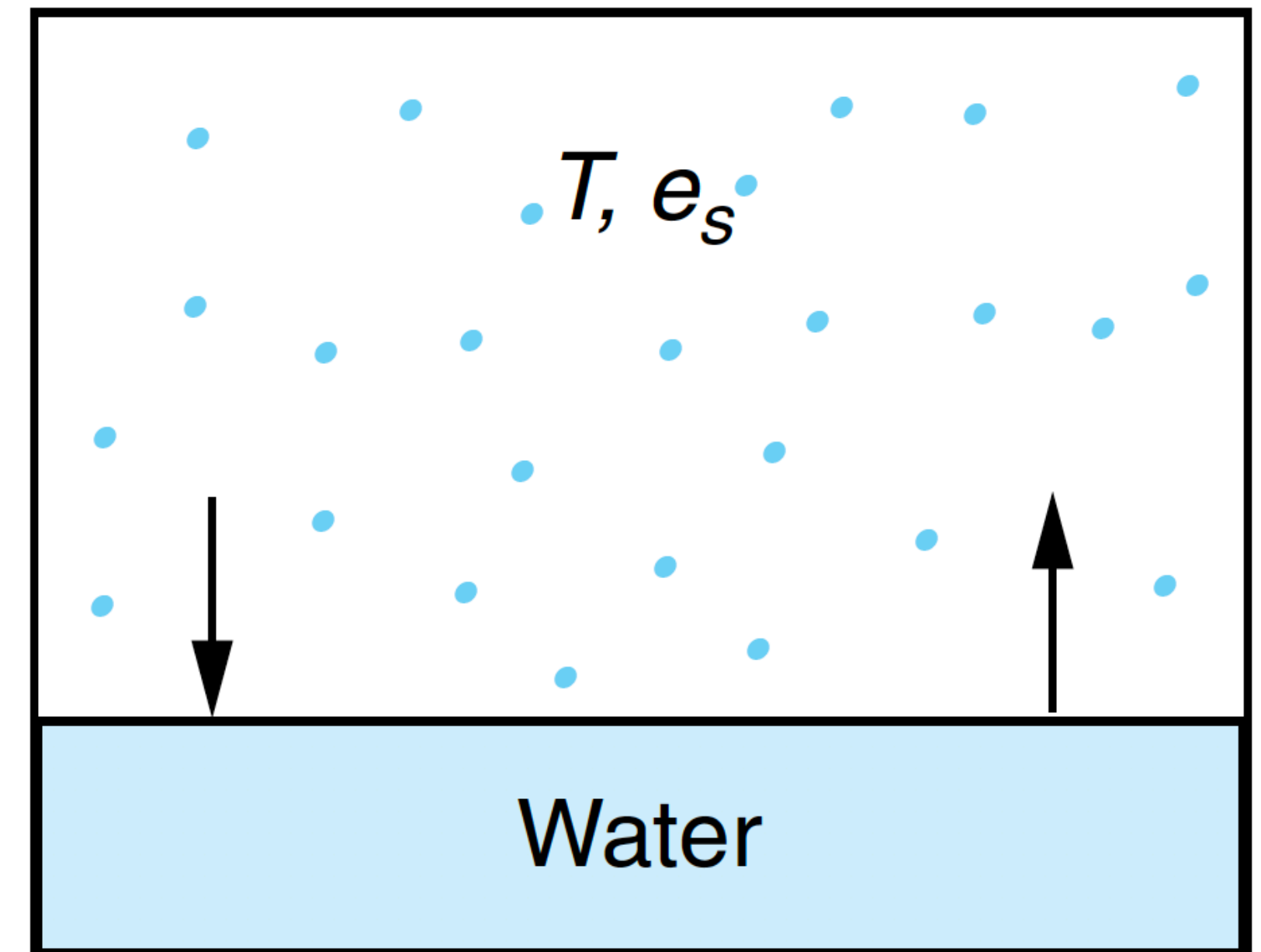
The vapor pressure for equilibrium is known as the saturation vapor pressure e_s

In this state of equilibrium, the tank satisfies

$$\frac{DG}{Dt} = 0$$

Which means that the change in G for evaporation and condensation must be equal

$$G_v = G_l$$

T, e_s
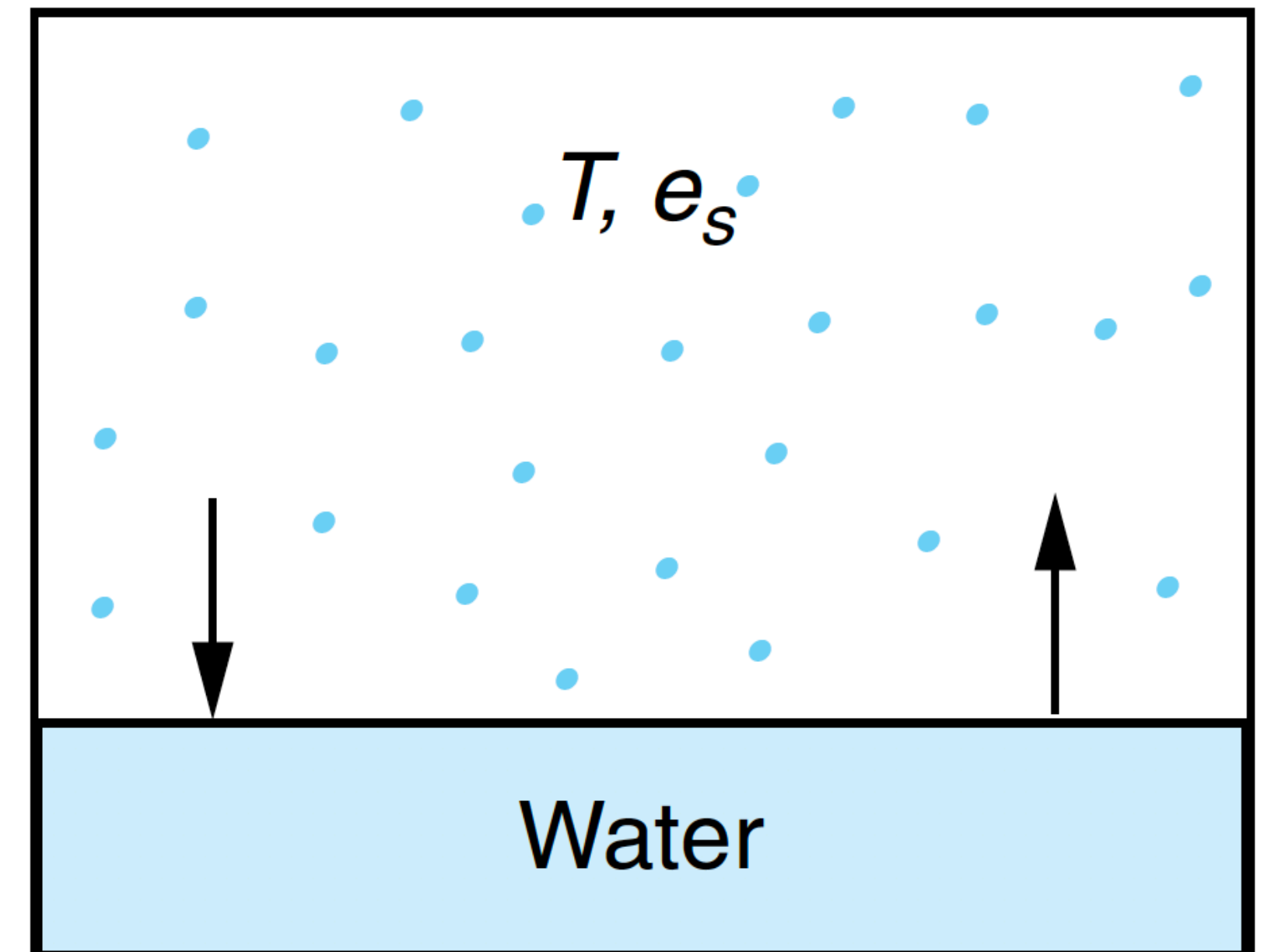
Water

(b) Saturated

Wallace and Hobbs (2006)

The vapor pressure for equilibrium is known as the saturation vapor pressure e_s

$$RH = \frac{e}{e_s}$$

If the RH is 100%, you have reached equilibrium. That is, the rate of evaporation to condensation is the same*

(*This only applies for flat surfaces of water. The story is different for cloud droplets)
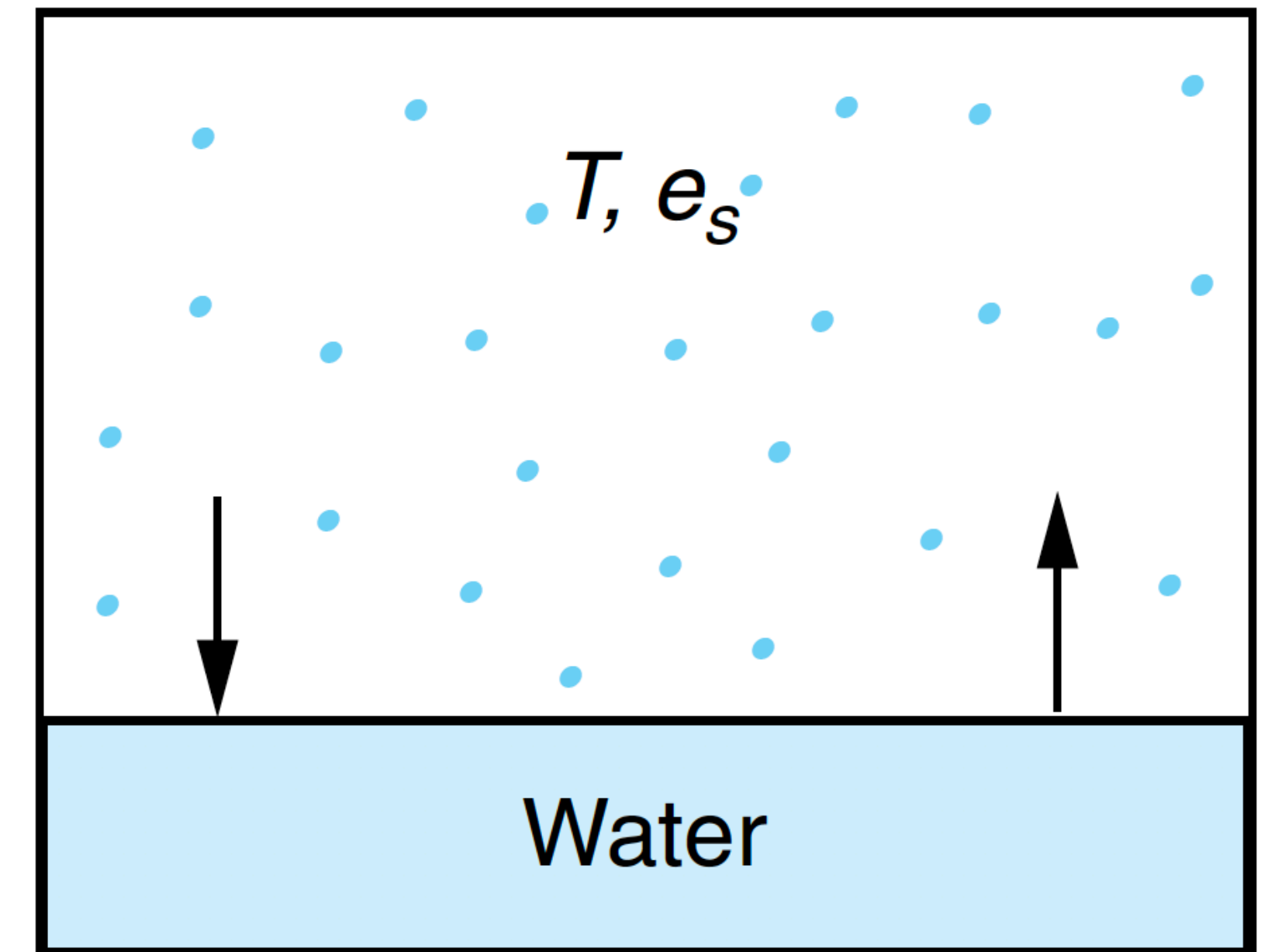

T, e_s

Water

(b) Saturated

Wallace and Hobbs (2006)

This means that if the tank is warmed for example, *G* in both systems must change equally when equilibrium is reached again.

$$dG_v = dG_l$$

Solving this equality and taking the limit yields the Clapeyron equation

$$\frac{de_s}{dT} = \frac{s_v - s_l}{\alpha_v - \alpha_l}$$

T, e_s

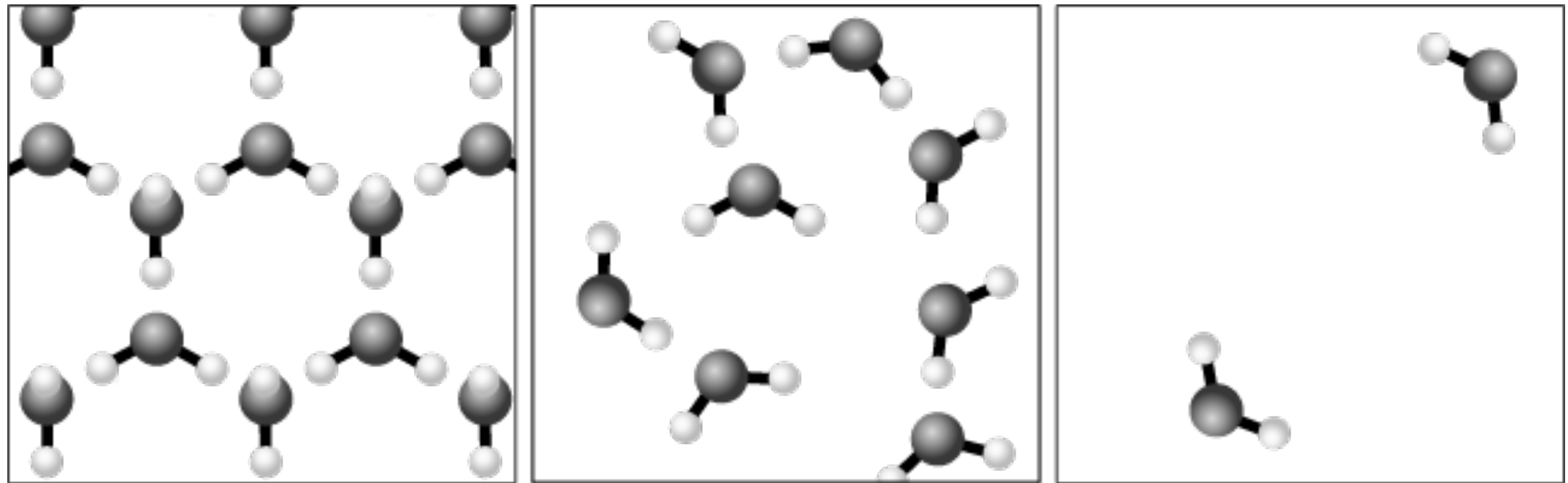Water

(b) Saturated

Wallace and Hobbs (2006)

Written in terms of entropy

$$L_v = T(s_v - s_l)$$

Physically-speaking, L_v describes the increase in enthalpy that the system obtains from the breakage of the molecular bonds in liquid.

If thought as a potential energy (PE), then you can think of the following conservation

$$(PE + h)_l = (PE + h)_v$$

Plugging the definition of L_v into the Clapeyron equation yields the Clausius-Clapeyron Equation.
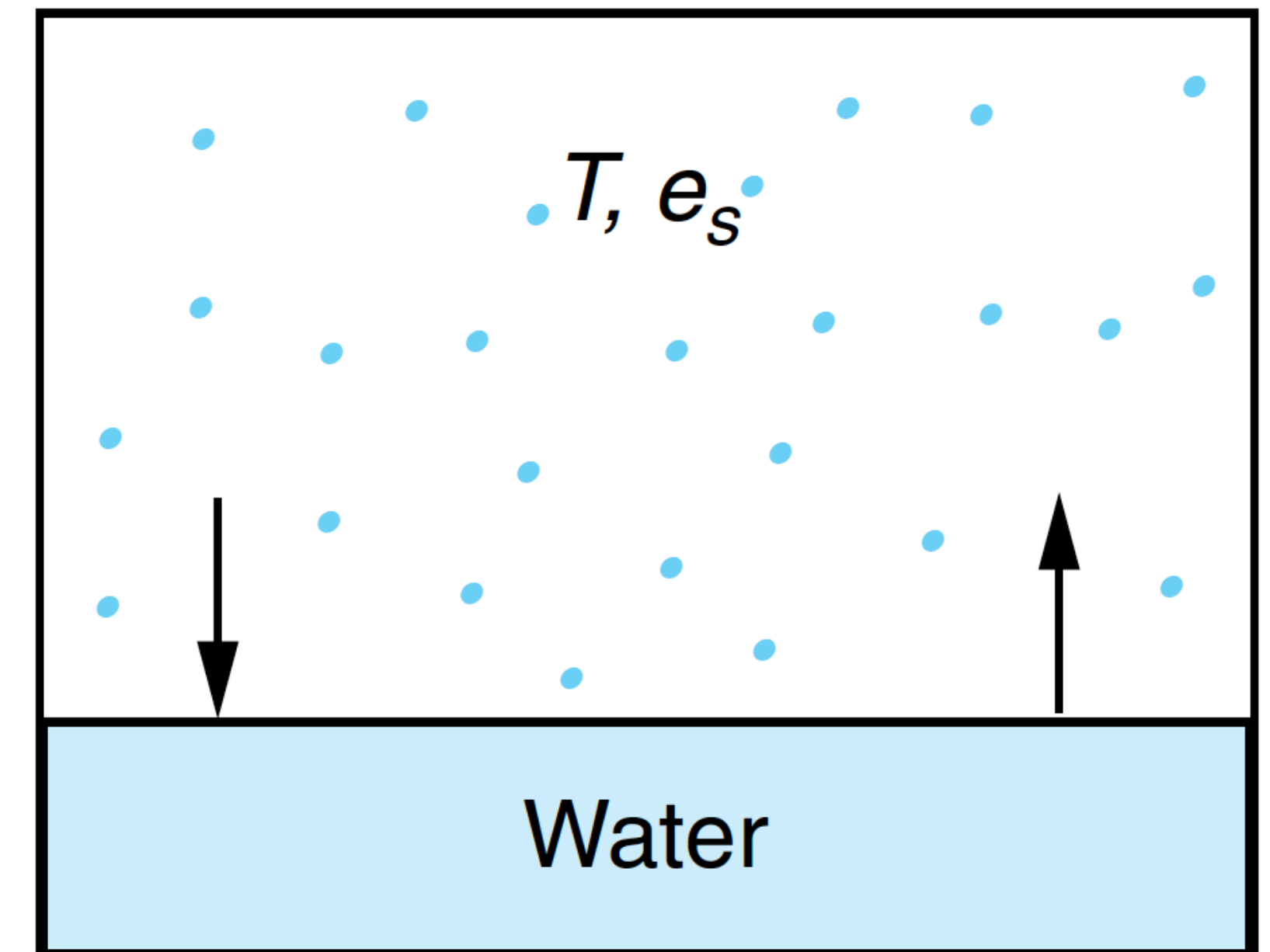
$$\frac{de_s}{dT} = \frac{L_v}{T(\alpha_v - \alpha_l)}$$

Assuming that

$$\alpha_v \gg \alpha_l$$

Yields a solution of the form

$$e_s \simeq e_{s0} \exp\left(\frac{L_v}{R_v}\left[\frac{1}{T_0} - \frac{1}{T}\right]\right)$$
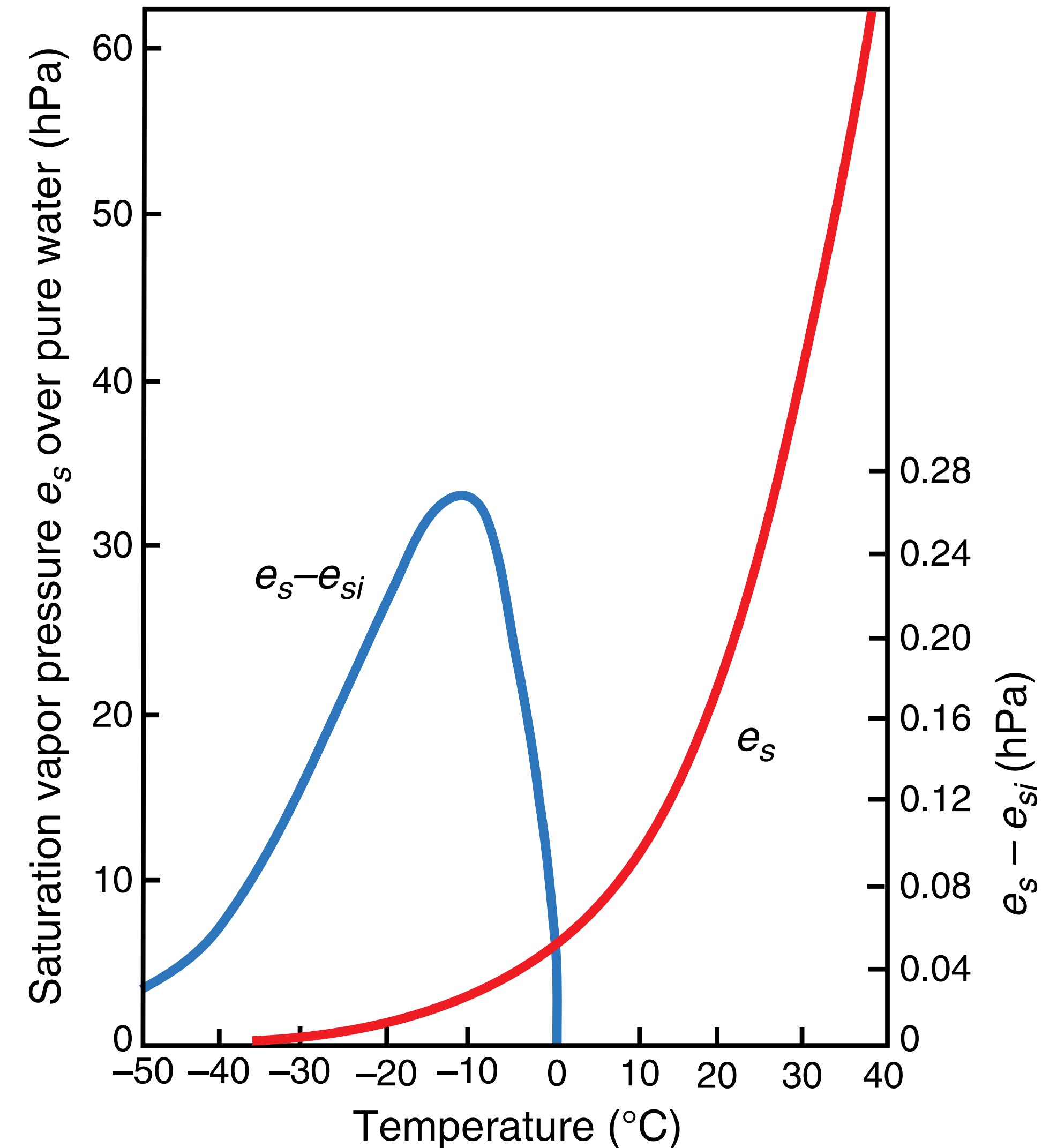

(b) Saturated

Wallace and Hobbs (2006)

A simplified version of the solution takes the following form

$$e_s = A \exp\left(-\frac{B}{T}\right)$$

$$A = 2.53 \times 10^9 hPa$$

$$B = L_v/R_v = 5.42 \times 10^3 K$$

This is a nice version that you can use to obtain other variables that are related to the Clausius-Clapeyron equation.

Wallace and Hobbs (2006)

Is the temperature in which an unsaturated air parcel will reach saturation if we adiabatically cool it and keep the mixing ratio and the pressure constant.

$$e(T_d) = e_s$$

Can use Clausius-Clapeyron to obtain it

$$T_d(p, q_v) = \frac{B}{\ln\left(\frac{A\varepsilon}{pq_v}\right)}$$

$$A = 2.53 \times 10^9 hPa$$

$$B = L_v/R_v = 5.42 \times 10^3 K$$

Saturation mixing ration and specific humidity

We can obtain saturation values of the mixing ratio and specific humidity

$$r_s = \varepsilon \frac{e_s}{p_d} \qquad\qquad q_s \simeq \varepsilon \frac{e_s}{p}$$
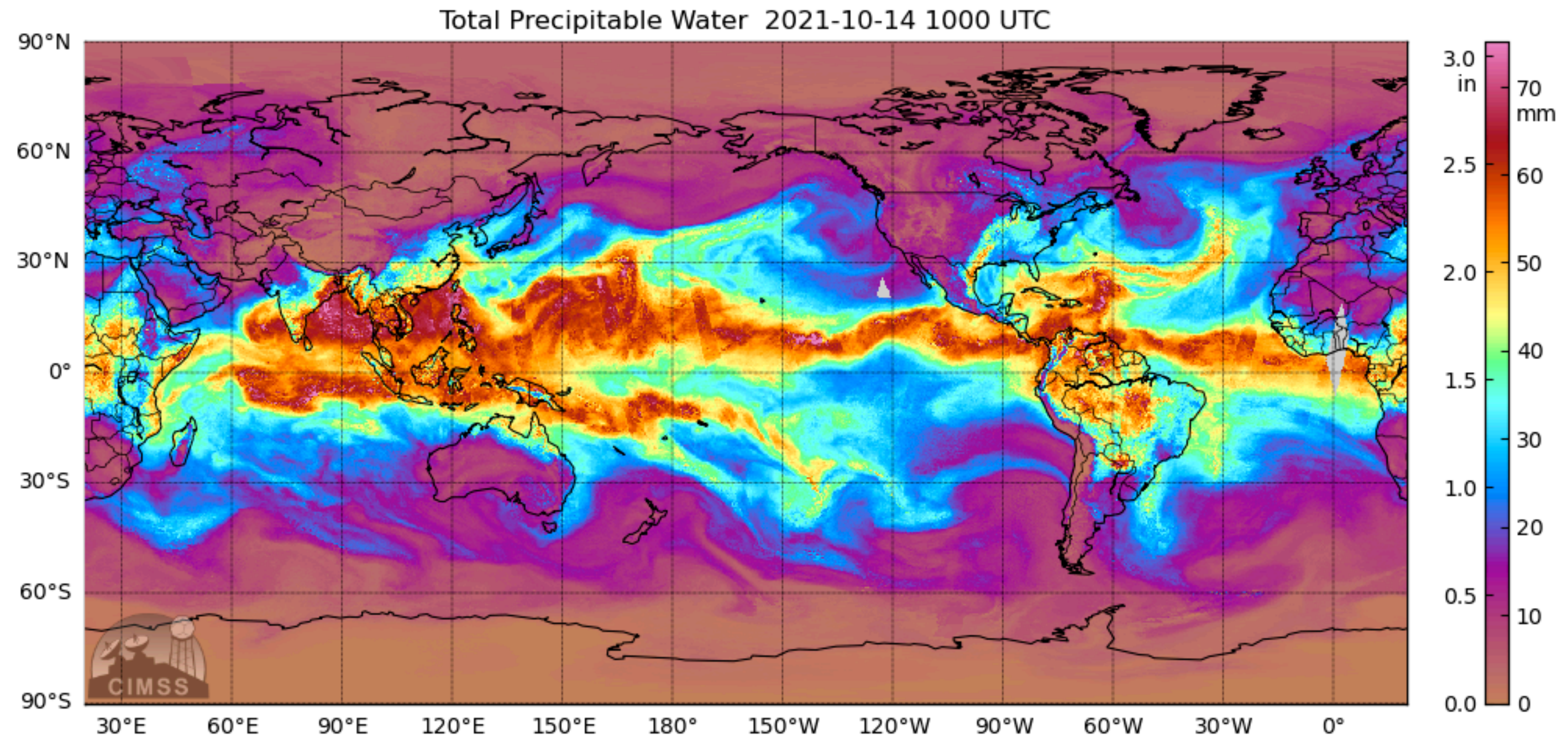
$$\varepsilon = R_d/R_v = 0.622$$

Same as the quantities defined previously, but for saturation vapor pressure

To a good approximation, saturation humidity scales with CC

$$\frac{dq_s}{dT} \simeq \frac{L_v}{R_v T^2}$$

$$\mathrm{RH} = \frac{q_v}{q_s}$$

Total Precipitable Water 2021-10-14 1000 UTC
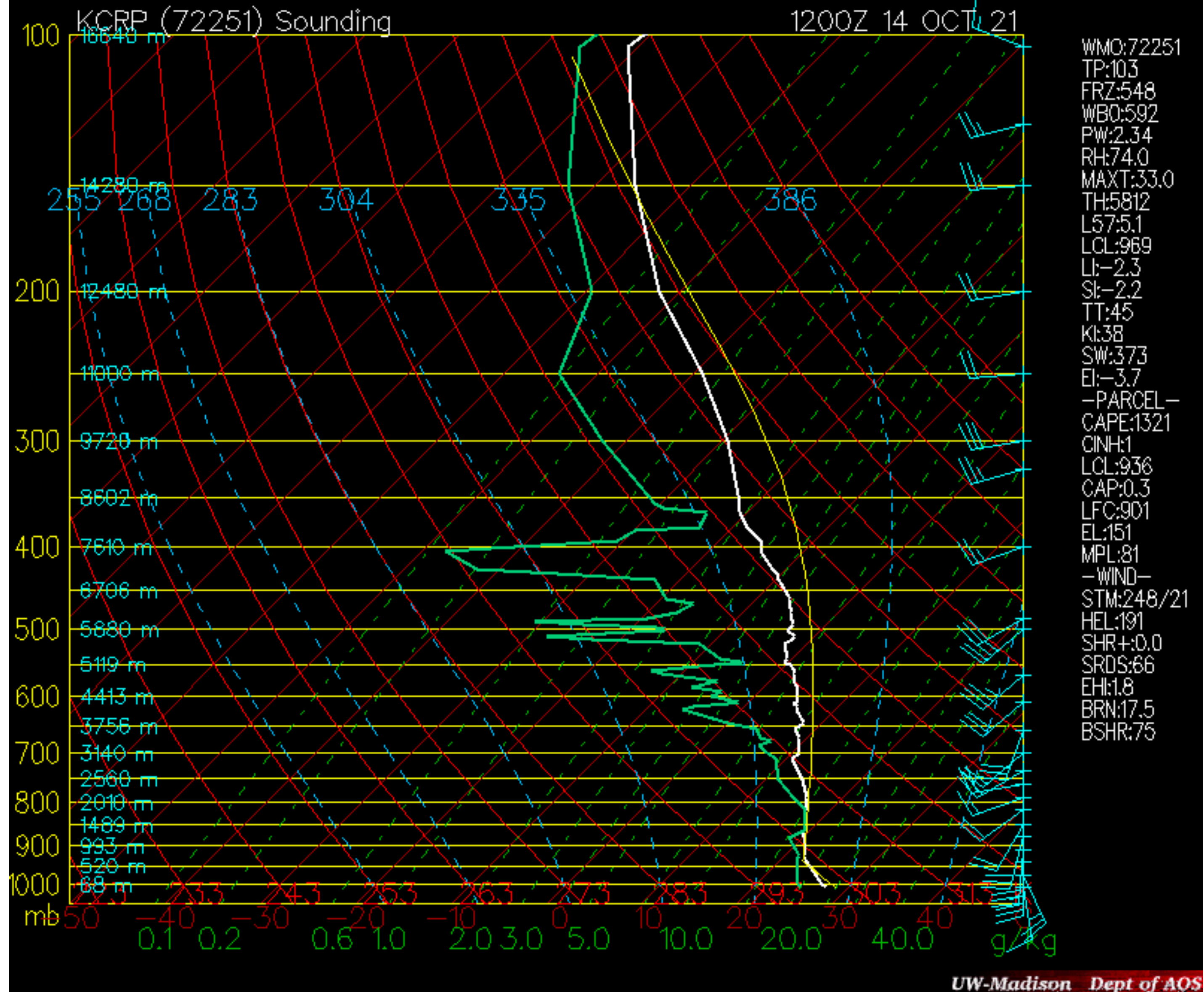
Dew Point depression

$$\Delta T_d = T - T_d$$
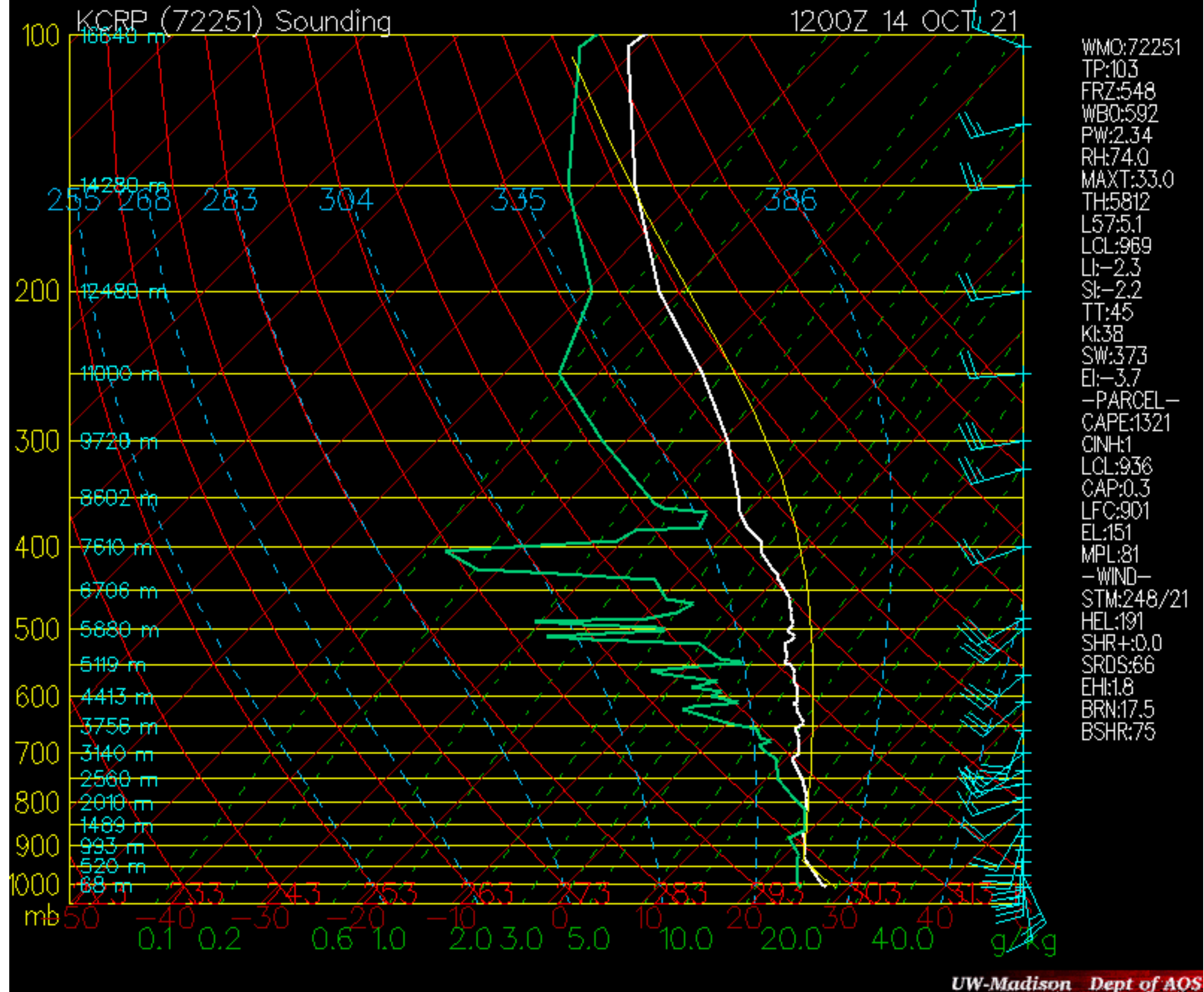
Separation between temperature and dew point

See now why the spacing for mixing ratio is not linear?

We are actually looking at the saturation mixing ratio.

RH vs dew point

The RH tells you how close you are to being saturated

$$RH = \frac{q_v}{q_s}$$

Dew point tells you the absolute value of moisture content.

They are different, but useful

GFS MSLP (mb), 700-400 hPa Relative Humidity (%), and 700-400 hPa Wind (kt)
Init: 12z Oct 14 2021   Forecast Hour: [6]   valid at 18z Thu, Oct 14 2021