Why does potential temperature (and hence entropy) increase in the atmosphere?

Approx. between θ and DSE (S_s)

$$S_s \simeq c_p\theta + \text{const}$$

$$c_pT + \Phi \simeq$$

$$c_pT + gz \simeq c_p\theta$$

"enthalpy" (c_pT)   potential energy (geopotential) (gz)

If gz increased at the same rate that c_pT decreased

$$c_p\frac{dT}{dz} + g \simeq 0 \longrightarrow \frac{dT}{dz} = -\frac{g}{c_p} = -\Gamma_d \quad \text{Dry adiabat}$$

In observations:

$$c_p\frac{dT}{dz} + g > 0$$

why? "Condensation in clouds keeps the lapse rate lower than dry adiabatic"

$$\frac{dT}{dz} > -\Gamma_d$$

*This is related to "radiative-convective" equilibrium
convection heats + radiation cools $\simeq 0$

2021 Nobel prize in physics.

Geopotential height:

$$gz = g_0 Z \quad \leftarrow \text{geopotential height}$$

(z: geometric height)

$g_0 = 9.8\ ms^{-2} \Rightarrow$ constant

$g \propto \frac{GMm}{r^2}$ = Newton's law of gravity
inverse square
weakens with square of distance from Earth

In practice $Z \simeq z$ because r = distance from Earth's center

$$r = a_{Earth} + z$$

(a_{Earth}: Radius of Earth; z: height above surface)

$$a_{Earth} \gg z$$

$r \simeq$ constant in the troposphere

$$g \simeq \frac{GMm}{a_{Earth}} \simeq g_0$$

"Approximating atmosphere as much shallower than Earth"

Why the different forms of the first law

Internal energy $c_v dT + p d\alpha = \delta q$ (1)

Enthalpy form $c_p dT - \alpha dp = \delta q$ (only for ideal gas!) (2)

(1) is convenient when $d\alpha = 0$ "isochoric process"

(2) is " when $dp = 0$ "isobaric process"

* the different forms are more or less applicable depending on what variable is staying fixed as well as what processes we know are occurring.