AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 7 Fall 2021

The Second Law of Thermodynamics

Ángel F. Adames-Corraliza
angel.adamescorraliza@wisc.edu

WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Announcements

HW1 is due today. Please upload to Canvas.

Second Skew-T a day was assigned last lecture. Due next Tuesday.

HW2 has been uploaded. Due October 14

Announcements

Next Thursday we will discuss the paper titled “Ocean temperatures chronicle the ongoing warming of Earth”, which the last problem in HW2 is based on.

Please read the paper before then so we can discuss openly.

You can solve HW2 Problem 4 before having this discussion.

Last Class : Adiabatic processes

When there is no diabatic heating (q=0), the system is *adiabatic*

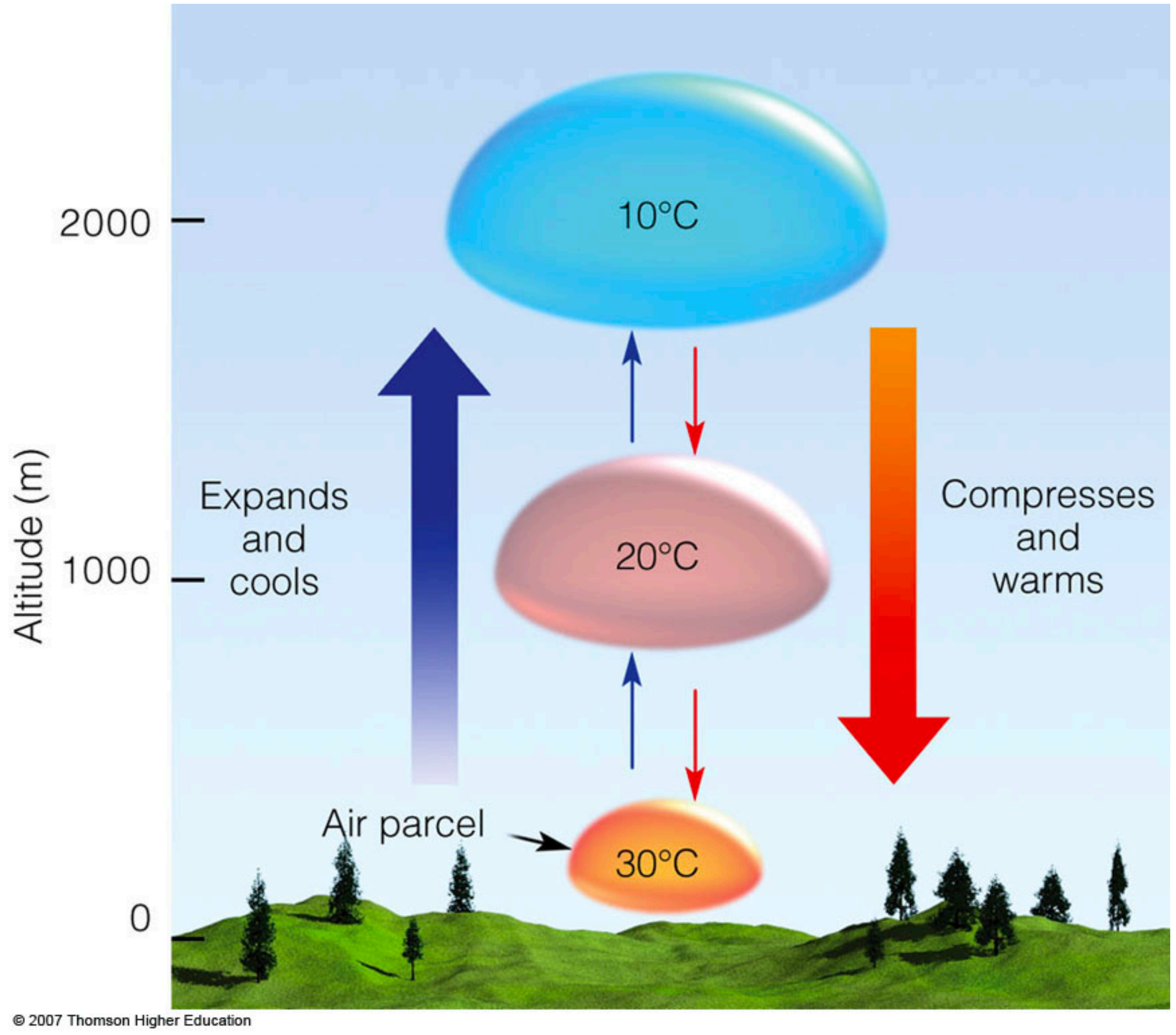
$$c_p \frac{dT}{dt} = \alpha \frac{dp}{dt}$$

We can solve equation to obtain:

$$\theta = T \left(\frac{p_0}{p} \right)^{R_d/c_p}$$

The potential temperature

The temperature a parcel would have if its adiabatically brought back to the surface.



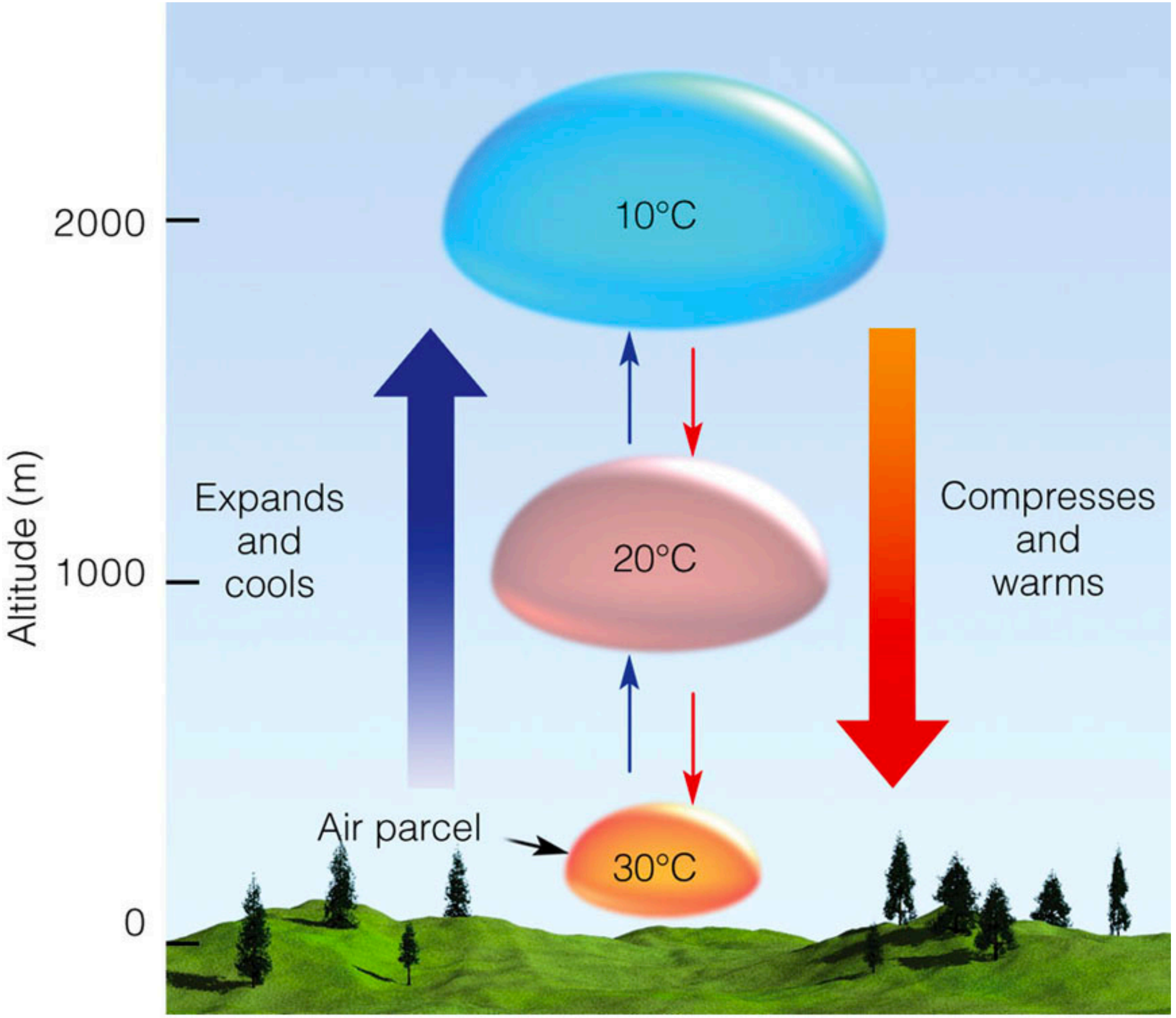
Dry static energy

A similar quantity to potential temperature is the dry static energy

$$s_s = c_p T + \Phi$$

The temperature a parcel would have if its adiabatically brought back to the surface.

The process needs to also be hydrostatic (vertical acceleration is small).

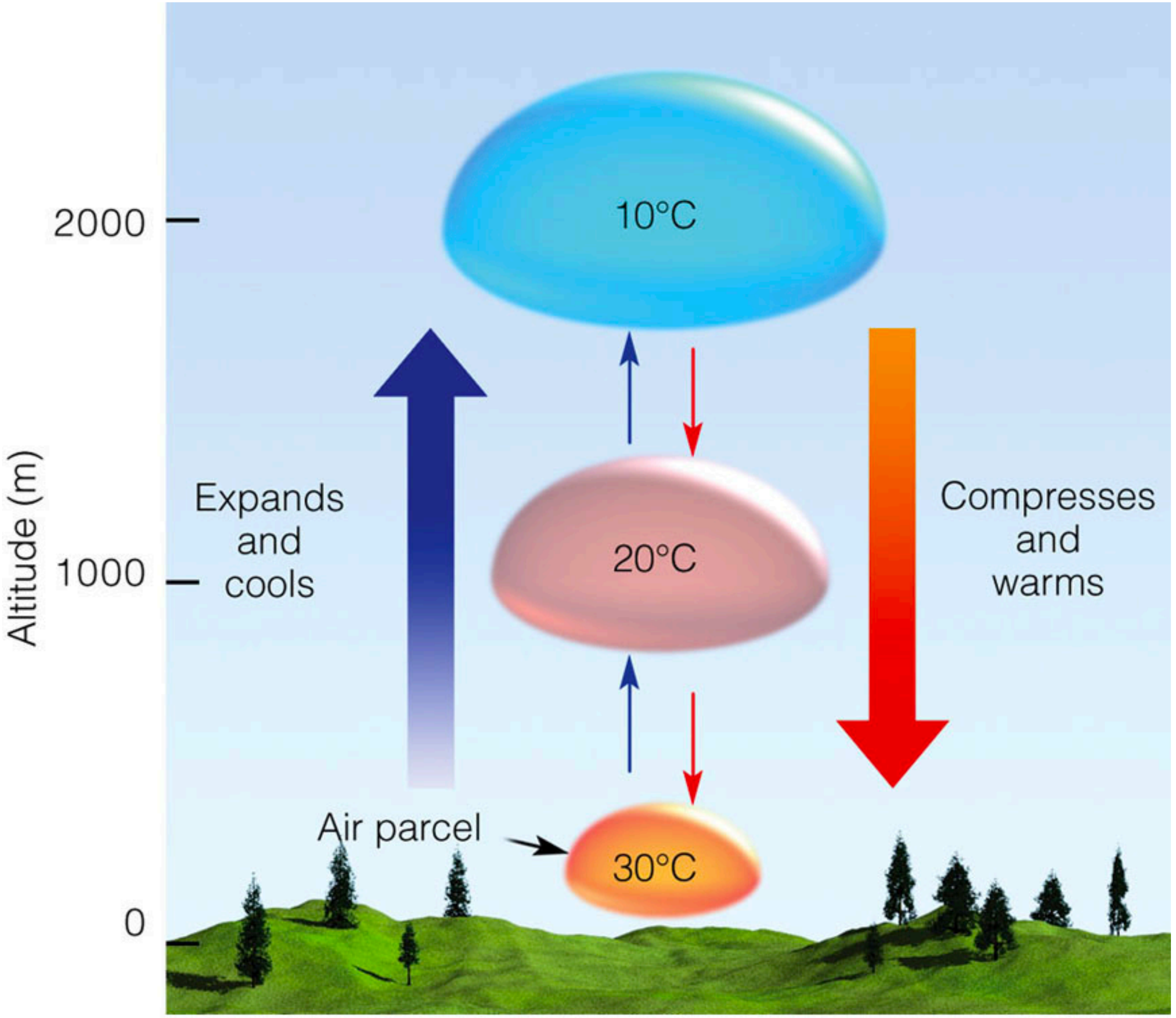


Dry adiabatic lapse rate

If a parcel is displaced vertically dry adiabatically, it would cool following the dry adiabatic lapse rate

$$\Gamma_d = \frac{g}{c_p}$$

$$\Gamma_d = 9.8\ \mathbf{K\ km}^{-1}$$

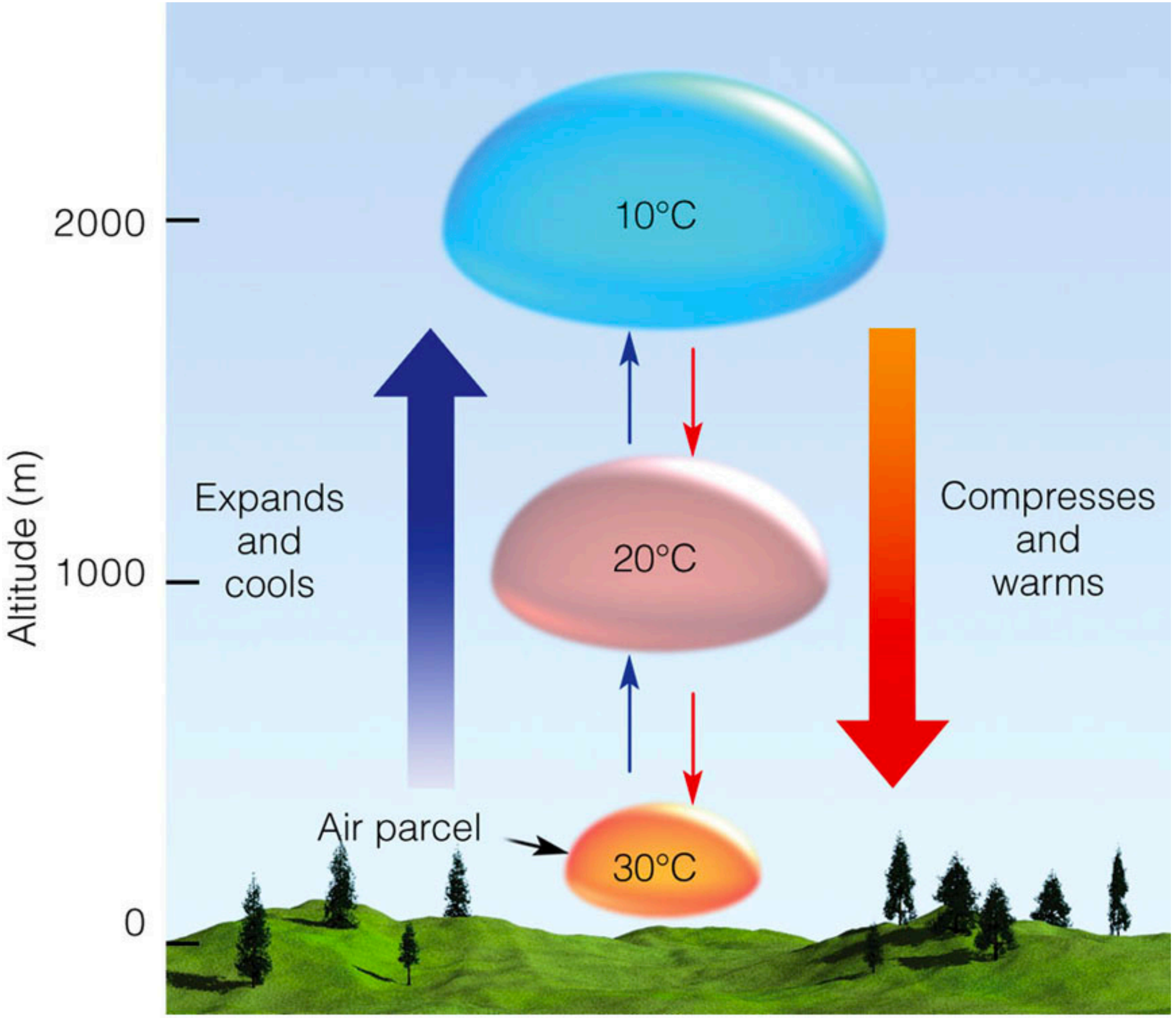


Why is the adiabatic lapse rate the way it is?

$$\Gamma_d = \frac{g}{c_p} \qquad \Gamma_d = 9.8\ \mathbf{K\ km}^{-1}$$

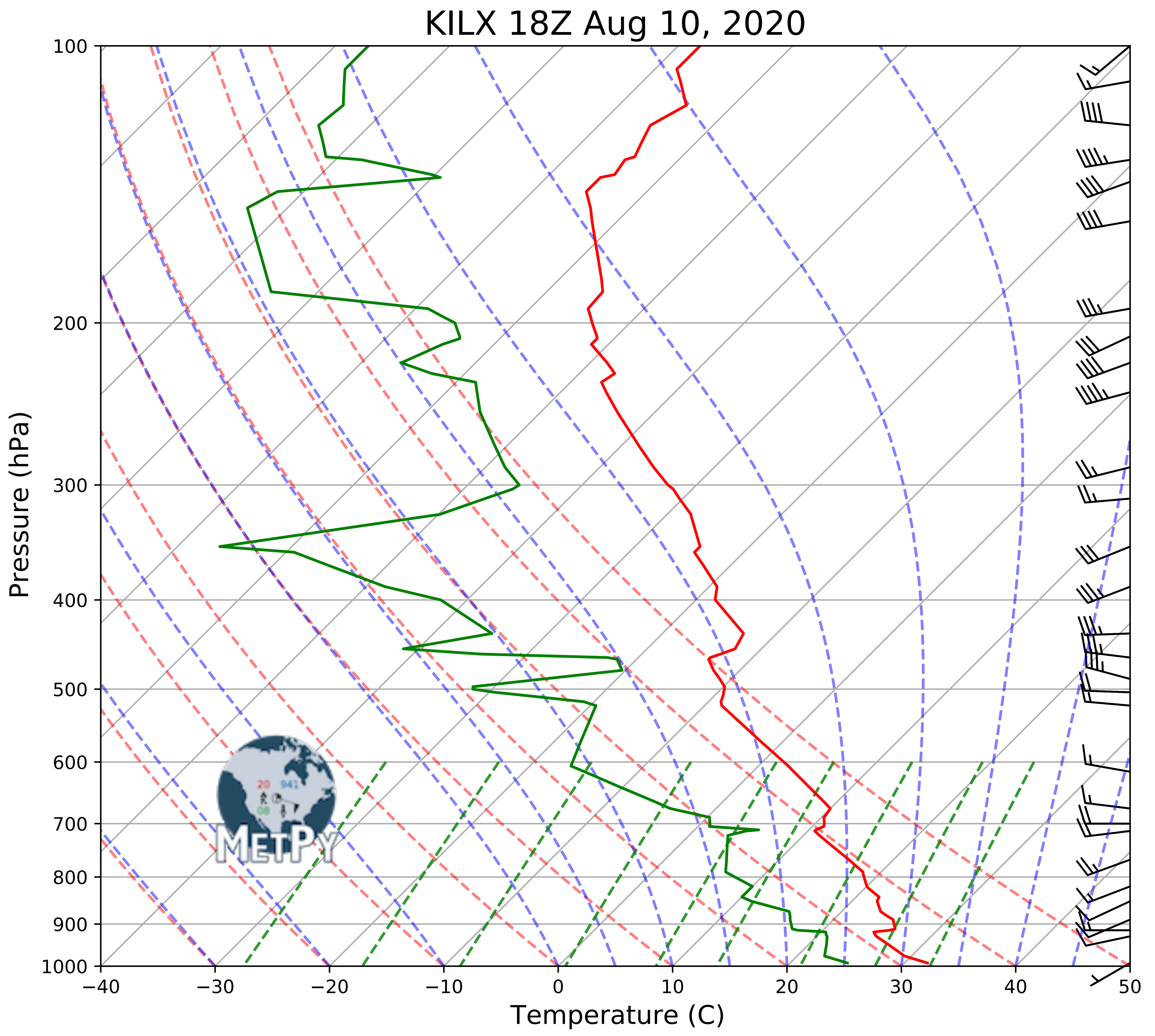
$$\frac{\partial s_s}{\partial z} = \frac{\partial}{\partial z}\left(c_p T + \Phi\right) = 0$$

Discuss with your colleagues for a few minutes



Dry adiabatic lapse rate on a Skew-T

Dashed red line in this diagram



Today

Finish discussion on the first law.

Begin discussing the second law of thermodynamics

Introduce **entropy** as a state variable.

Dry static energy and potential temperature

In general (not assuming adiabatic processes), the potential temperature and DSE have the following equations

$$\frac{D\theta}{Dt} = \frac{\theta\dot{Q}}{c_p T}$$

Potential temperature form of the first law

$$\frac{Ds_s}{Dt} = \dot{Q}$$

DSE form of the first law

DSE and potential temperature are related by the following formula

$$ds_s \simeq c_p T d\ln\theta$$

The second law of thermodynamics

Reversible processes

Processes in which the system (i.e. a parcel) is always in thermodynamic equilibrium with the environment.

For a process to be reversible, it must occur very slowly, so that the system has time to adjust to an equilibrium state.

In reversible processes, undoing the process (i.e. doing it backwards after you've finished it) would lead to exactly the same state where you started.

Example: bring a parcel upward dry adiabatically and then bring it back down.

In contrast, an irreversible process is one that you can't revert back to the initial state.

In general, irreversible processes are characterized by some form of change in the heat content by the end of the process

$$\delta q > 0$$

Example. You bring a parcel up, some of water vapor condenses and rains out. The water molecules have left the parce. When you bring the parcel back down it will not be the same its initial state.

Entropy

We can define a quantity known as entropy to understand is a process is reversible or not

$$ds = \frac{\delta q}{T}$$

The word "entropy" comes from the greek word for transformation.

In reversible processes $ds = 0$

In irreversible processes $ds > 0$

Note the exact differential. **Entropy is a state variable.**

Entropy form of the first law

The first law of thermodynamics can be written in terms of entropy as:

$$ds = c_v d\ln T + R_d d\ln \alpha$$

Entropy form of the first law.

We can use the ideal gas law to write it in this form (useful for the atmosphere)

$$ds = c_p d\ln T - R_d d\ln p$$

Entropy form of the first law, using c_p.

Can write in material derivative form by replacing *d* with *D/Dt.*

Entropy and potential temperature

Entropy is directly related to the natural log of potential temperature

$$ds = c_p d \ln \theta$$

Entropy-theta form of the first law.

Which means that **potential temperature is a measure of entropy!**

Can solve to obtain the following

$$s = c_p \ln \theta + \text{constant}$$

Entropy and dry static energy

We can use the two relationships below to relate the entropy to the DSE

$$ds = c_p d \ln \theta \qquad\qquad ds_s \simeq c_p T d \ln \theta$$

Which yields

$$T ds = ds_s$$

Entropy-DSE form of the first law.

What is entropy anyway?

Entropy can be thought in terms of **multiplicity.**

Multiplicity: number of ways that you can arrange the constituents (i.e. atoms) of a system in order to get an observed large-scale state (the macro state)

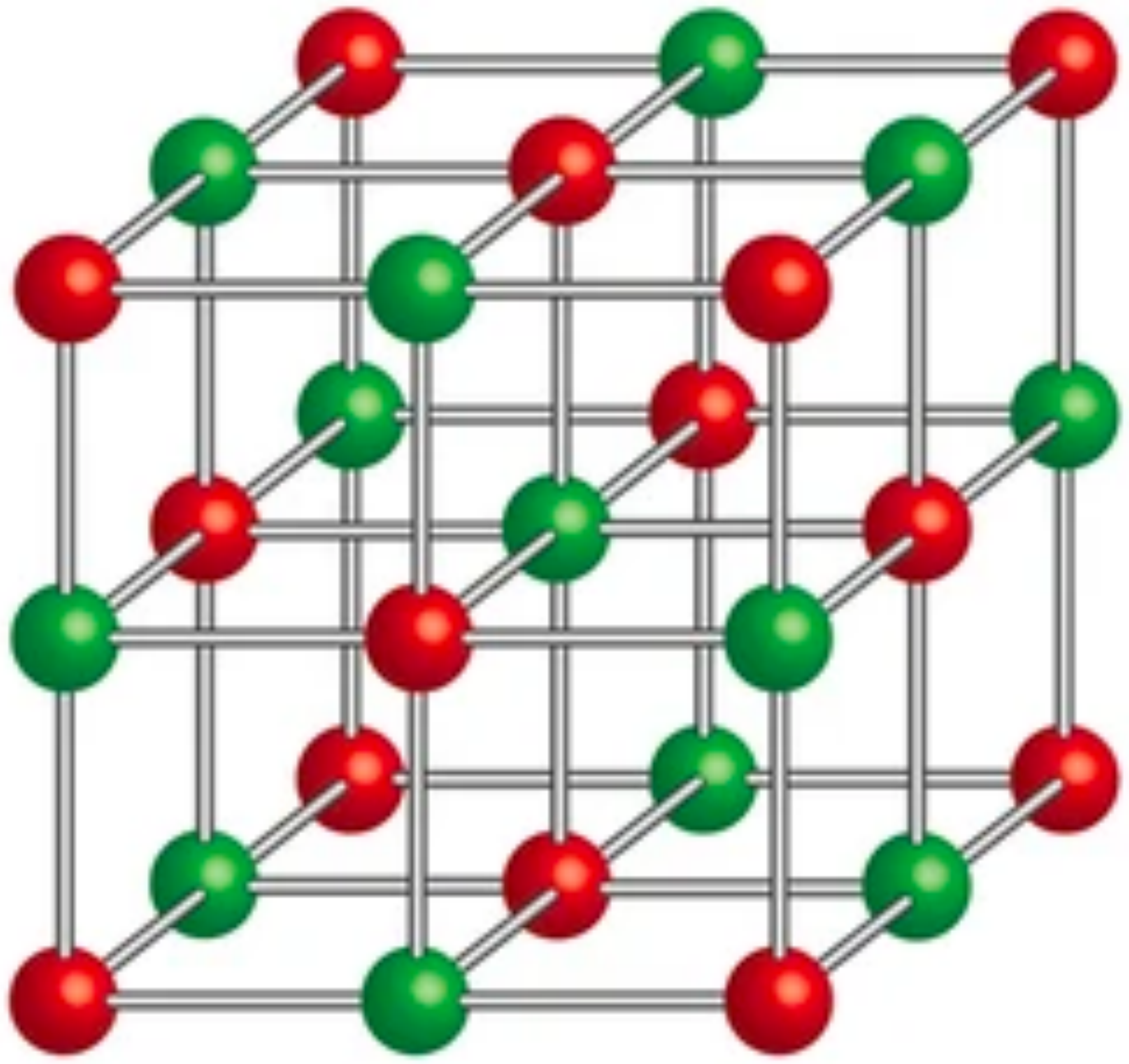
From multiplicity is where we get the expression that entropy “is a measure of disorder”

What is entropy anyway?

Low Entropy

Low Multiplicity

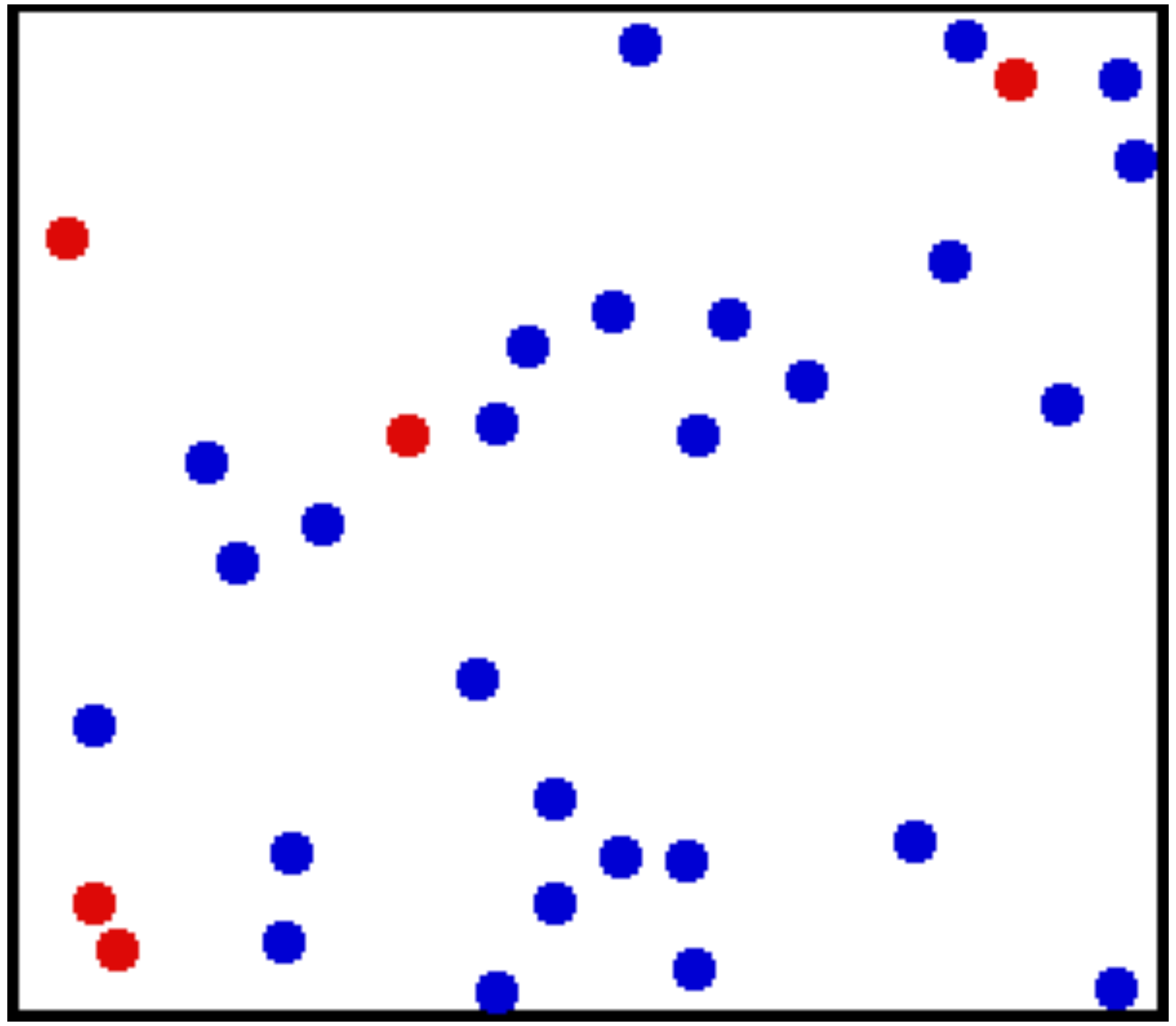
There's only a limited amount of ways in which you can rearrange the atoms in this lattice and still have a lattice



High Entropy

High Multiplicity

The molecules can rearrange themselves freely and still be a gas.



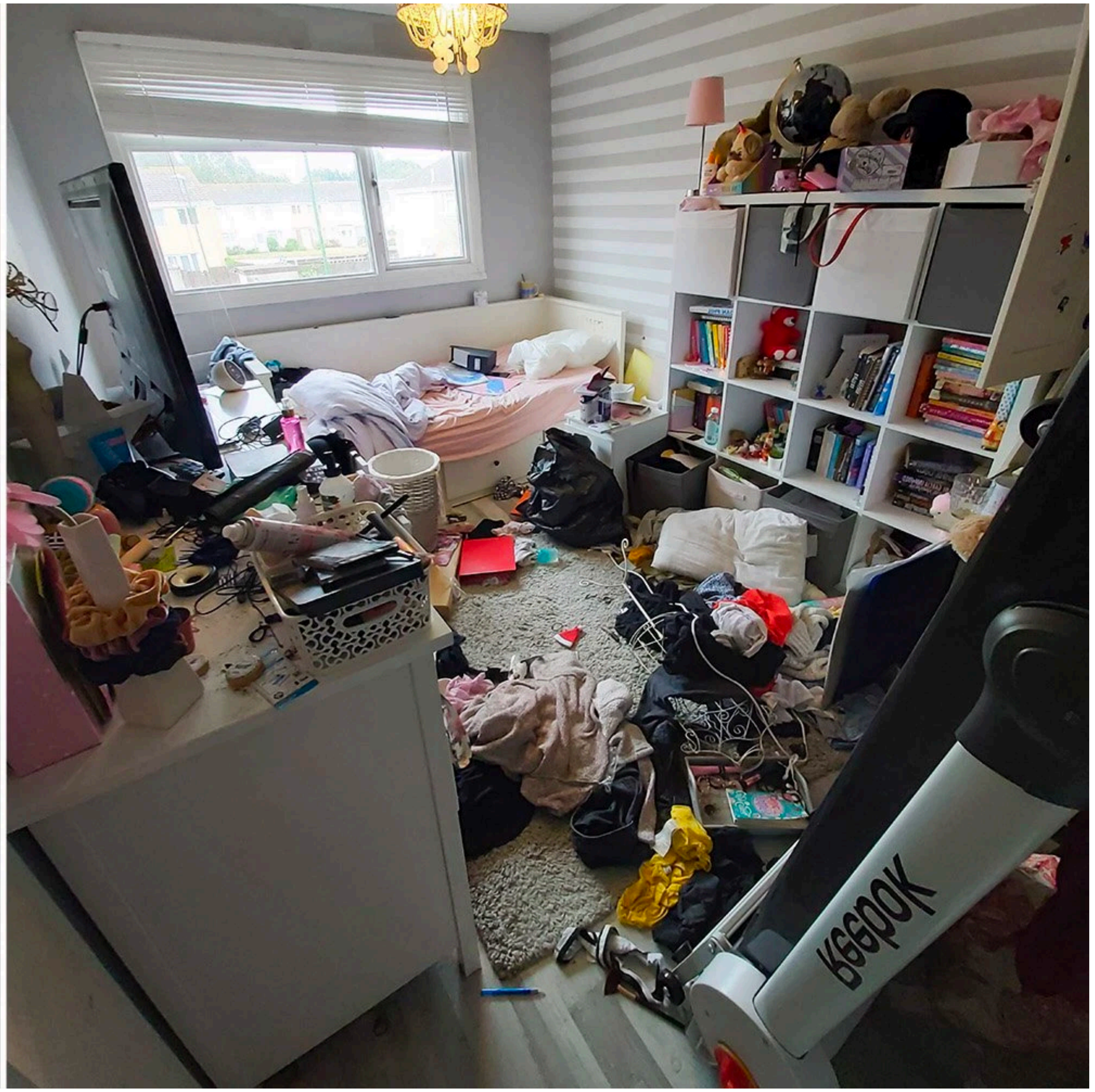
What is entropy anyway?

Low Entropy

Low Multiplicity

High Entropy

High Multiplicity



Entropy and multiplicity

Multiplicity: number of ways that you can arrange the constituents (i.e. atoms) of a system

$$ds = k_B d \ln \mu$$

$$\mu = \text{multiplicity}$$

$$k_B = \text{Boltzmann Constant}$$

$$k_B = 1.38 \times 10^{-23} \text{ J kg K}^{-1}$$

We can write the potential temperature in terms of the multiplicity:

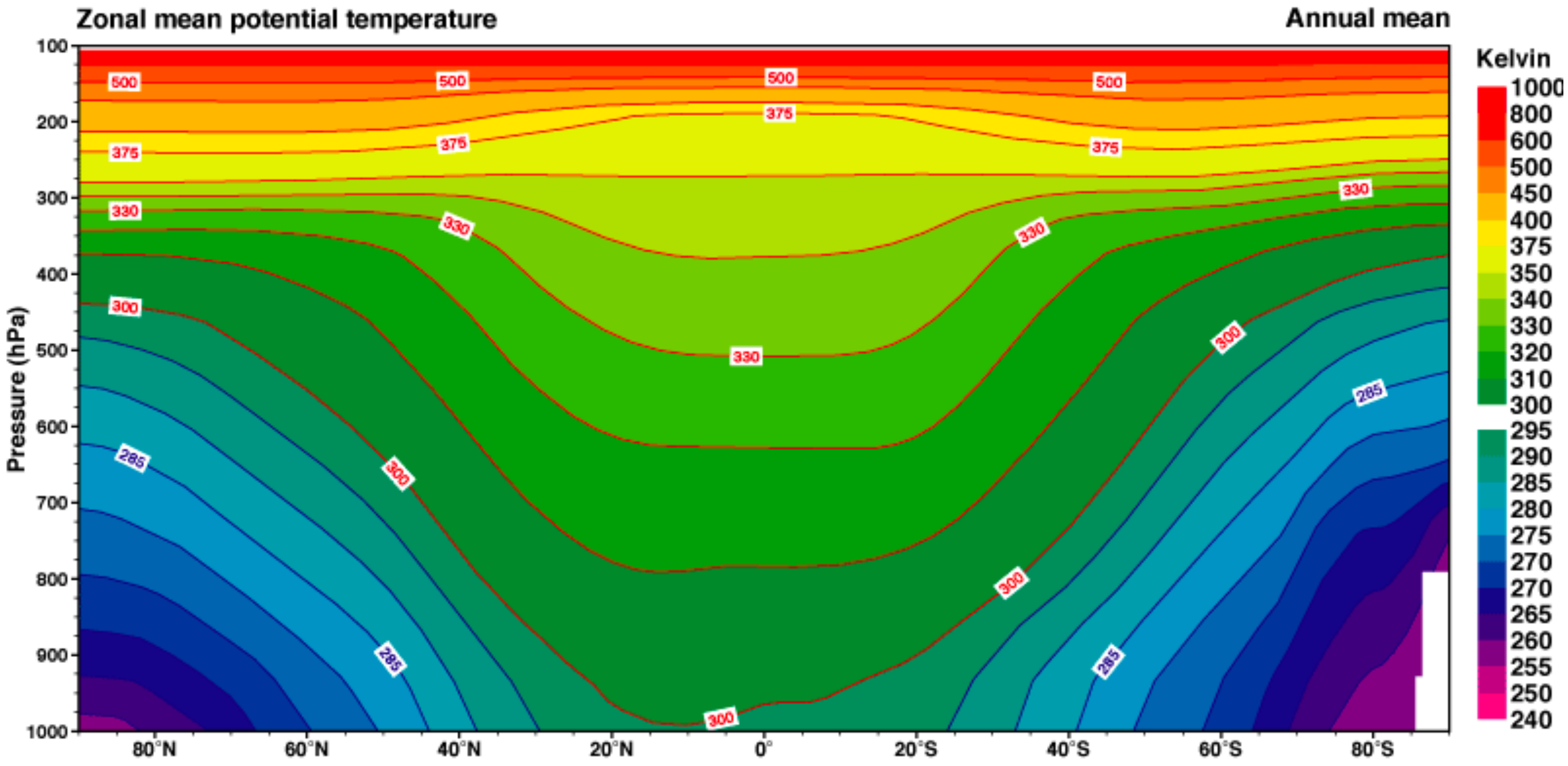
$$c_p d \ln \theta = k_B d \ln \mu$$

Changes in potential temperature are related to changes in the multiplicity of the system.

Exercise

Discuss why potential temperature (hence entropy) increases with height in the atmosphere. Use dry static energy and multiplicity to make your case.

Hint: Think about the density and specific volume of air as height increases. Remember that we have defined our system in **intensive** form.



Remark

Slow, equilibrated processes are incredibly rare. Our universe is characterized by spontaneous processes.

As a result, most processes that we observe are **irreversible.**

Entropy is always increasing.

The first law of thermodynamics

Law: Energy is conserved everywhere (system + environment)

Consequence: The change in the energy of a system is due to an exchange between it and its surrounding environment.

The second law of thermodynamics

Law: Within any system that is not in thermodynamic equilibrium with its environment entropy **must** increase. Equilibrium is achieved when entropy reaches its highest value.

Consequence: The universe we live in is constantly evolving, and thermodynamic equilibrium is rarely achieved within it. Thus ***entropy is always increasing***.