The first law of thermodynamics:

$$(1) \qquad c_v \frac{DT}{Dt} = \dot{Q} - p\frac{D\alpha}{Dt}$$ "Internal energy form of 1st law"

Note: $\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dx}{dt}\frac{\partial}{\partial x} + \frac{dy}{dt}\frac{\partial}{\partial y} + \frac{dz}{dt}\frac{\partial}{\partial z}$

3-D flow Knowing that $T = T(x,y,z,t)$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

where u, v, and w are the 3-D wind components

* the 1st law follows parcels as they move with the flow

We can rewrite the first law by using the chain rule and plug into Eq. (1)

$$\frac{D(p\alpha)}{Dt} = p\frac{D\alpha}{Dt} + \alpha\frac{Dp}{Dt}$$

$$p\frac{D\alpha}{Dt} = \frac{D(p\alpha)}{Dt} - \alpha\frac{Dp}{Dt}$$

Back to 1st law: $c_v\frac{DT}{Dt} = \dot{Q} - \left[\frac{D(p\alpha)}{Dt} - \alpha\frac{Dp}{Dt}\right]$

$$c_v\frac{DT}{Dt} + \frac{D(p\alpha)}{Dt} = \dot{Q} + \alpha\frac{Dp}{Dt}$$

Define the enthalpy $h = c_vT + p\alpha$ and plug into 1st law:

$$\frac{Dh}{Dt} = \dot{Q} + \alpha\frac{Dp}{Dt}$$

$p\alpha = R_dT$ ideal gas

Enthalpy of an ideal gas $h = c_vT + R_dT$
$= (c_v + R_d)T$
$= c_pT$

$c_p = c_v + R_d$

Why do we have c_p and c_v

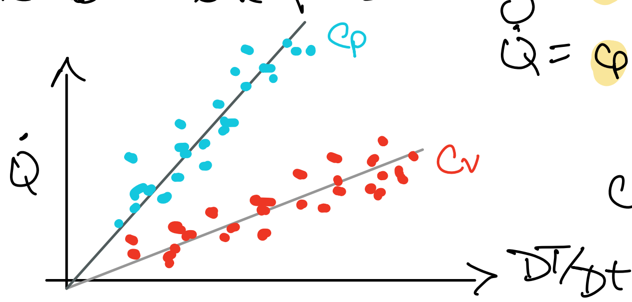
$$c_v\frac{DT}{Dt} = \dot{Q} - \cancel{p\frac{D\alpha}{Dt}}$$

isochoric process (V = constant, α = const)

$$c_p\frac{DT}{Dt} = \dot{Q} + \cancel{\alpha\frac{Dp}{Dt}}$$

isobaric process (p = constant)

Think about line plots



$y = mx$

$\dot{Q} = c_p\frac{DT}{Dt}\Big|_{p=\text{const}}$ $\qquad \dot{Q} = c_v\frac{DT}{Dt}\Big|_{v=\text{const}}$

$c_p = c_v + R_d$

Other types of thermo. processes

$$\cancel{c_v \frac{DT}{Dt}} = \dot{Q} - p\frac{D\alpha}{Dt}$$ isothermal process (T = fixed)

$$c_v \frac{DT}{Dt} = \cancel{\dot{Q}} - p\frac{D\alpha}{Dt}$$ adiabatic process ($\dot{Q} = 0$)

The adiabatic process is particularly important for the atmosphere.

The first law (in enthalpy form), with "no time"

ideal gas: $p\alpha = R_d T$, $\alpha = \frac{R_d T}{p}$

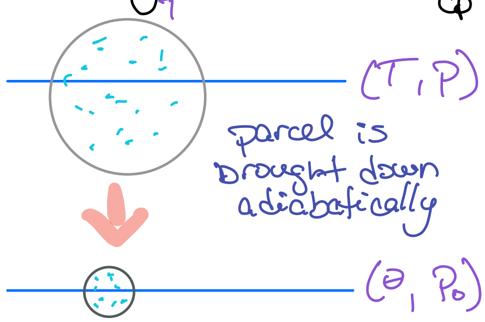
$$c_p dT - \alpha dp = 0$$ adiabatic process

$$c_p dT - \frac{R_d T}{p} dp = 0$$ rearranging

$$\frac{dT}{T} - \frac{R_d}{c_p}\frac{dp}{p} = 0$$ using ln identities:

$$d\ln T - \frac{R_d}{c_p} d\ln p = 0$$

$$\int_T^{\theta} d\ln T = \frac{R_d}{c_p}\int_p^{p_0} d\ln p$$



swap integral limits

$$\int_{\theta}^{T} d\ln T = \frac{R_d}{c_p}\int_{p_0}^{p} d\ln p$$

Solving the integral yields:

$$\ln\frac{\theta}{T} = \frac{R_d}{c_p}\ln\frac{p_0}{p}$$

$$\ln\frac{\theta}{T} = \ln\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}$$

Exponential on both sides yields:

$$\frac{\theta}{T} = \left(\frac{p_0}{p}\right)^{R_d/c_p}$$

Rearranging: $$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}$$

potential temperature

Usually $p_0 = 1013$ hPa sfc pressure

Another quantity similar to θ is known as the dry static energy (DSE, s_s)

Returning to "enthalpy" form of 1st law:

$$c_p dT - \alpha dp = 0$$

In a hydrostatic atmosphere $\frac{\partial p}{\partial z} = -\rho g \longrightarrow \alpha dp = -g dz = -d\Phi$

Rewriting $c_p dT + d\Phi = 0$ Define $s_s = c_p T + \Phi$ DSE

$$ds_s = 0$$

DSE is like θ but assuming hydrostatic balance

in temporal form $\frac{Ds_s}{Dt} = 0$ for adiabatic in 1-D atmosphere (z coord only)

$$\frac{Ds_s}{Dz}\frac{Dz}{Dt} = 0$$

$$w\frac{Ds_s}{Dz} = 0$$

$$\frac{d}{dz}\left(c_p T + gz\right) = 0$$

$$c_p \frac{dT}{dz} + g = 0$$

$$\frac{dT}{dz} = \frac{-g}{c_p} = -\Gamma_d$$

$\Gamma_d = \frac{g}{c_p}$ dry adiabatic lapse rate