AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 5 Fall 2021

The First Law of Thermodynamics

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Announcements

HW1 problem 6: The mass per unit area of any given chemical substance can be obtained from

$$m_x = \int_{z_0}^{z_1} \rho_x dz$$

Where x is any given substance.

For the last part of the problem note that column-integrated water vapor (CWV) has units of kg/m^2, and the loops on the website are in mm (millimeters). It turns out that 1 kg/m^2 of CWV = 1 mm CWV.

Also note that you need a surface pressure value of p0 = 1013 hPa.

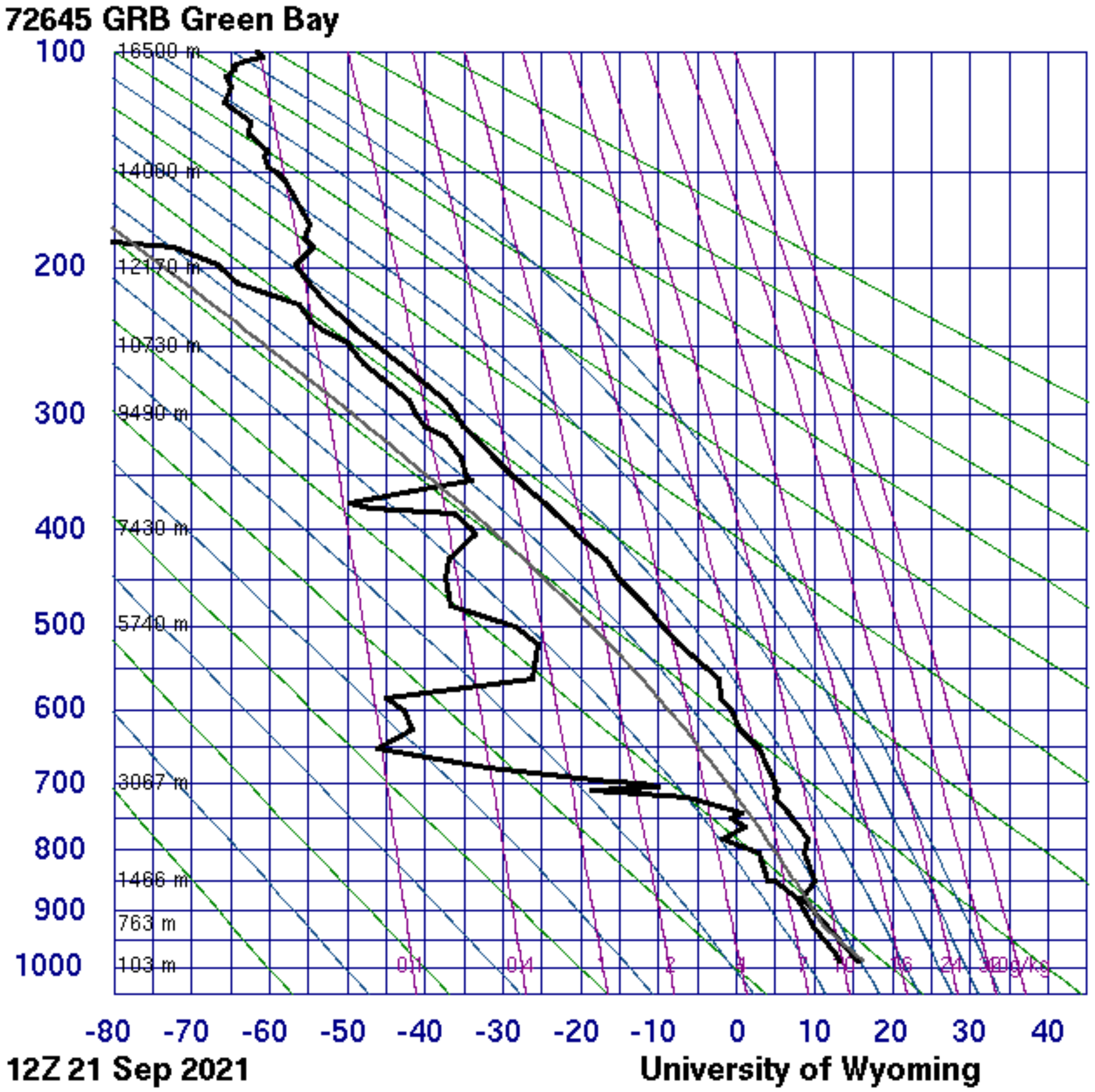
Announcements

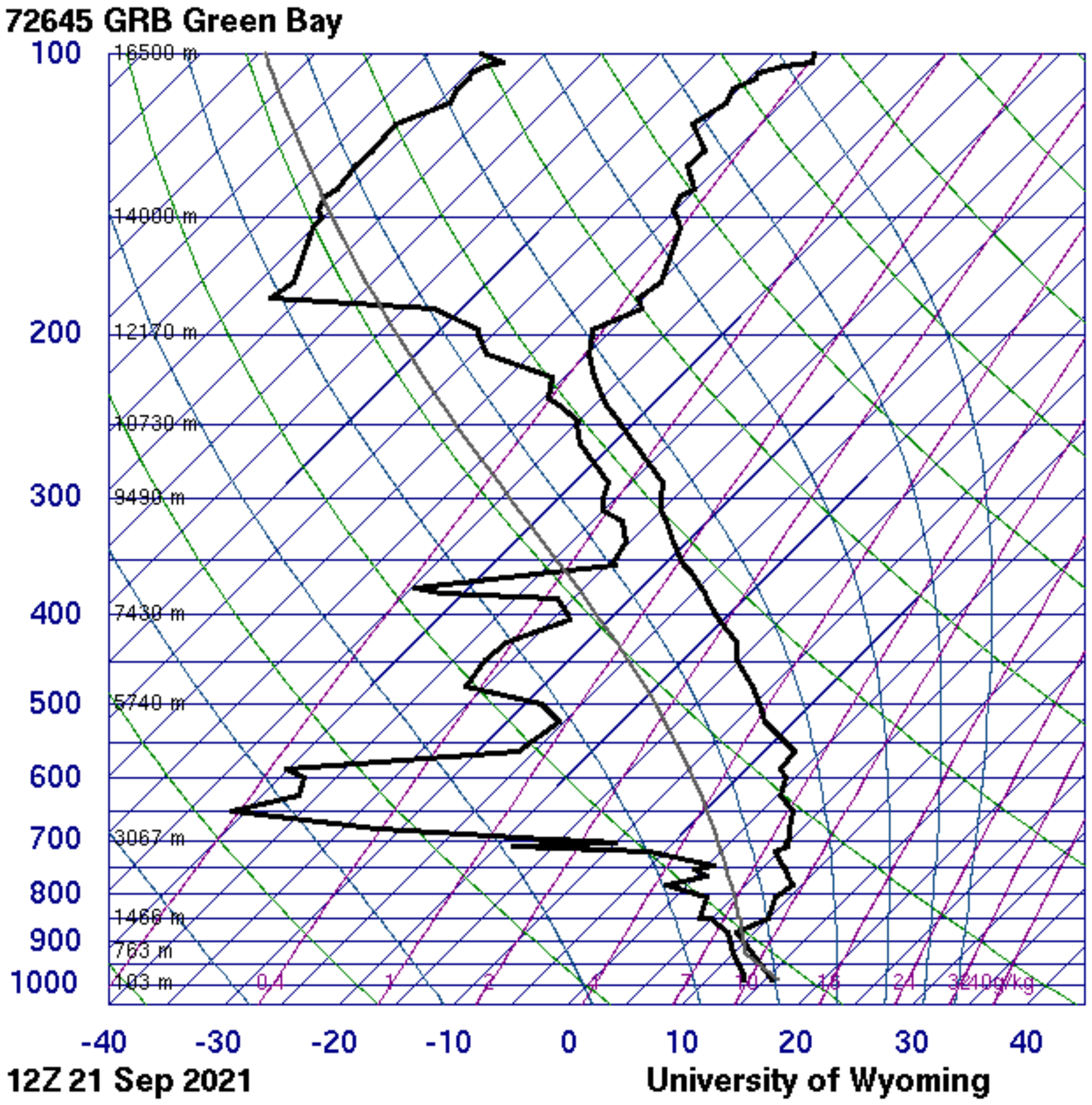
Skew-T a week 1 is due next Tuesday. Should be uploaded to Canvas.

Homework 1 is due one week from today. Should be uploaded to Canvas.

Last class: Stuves and SkewT

A Stuve is like an emagram but with more stuff





Skew-T a week #1

Download two Skew-T diagrams from two different locations (recommend far away from each other).

1. Discuss the temperature profiles in them.

2. What kind of lapse rate do you see?

3. What can we say about temperature profiles in the atmosphere?

Attach the soundings and discussion of the three points above and upload it to Canvas. It is due one next Tuesday.

Useful Links:
https://www.aos.wisc.edu/weather/wx_obs/Soundings.html
http://weather.uwyo.edu/upperair/sounding.html

Today

Begin discussion on the first law of thermodynamics

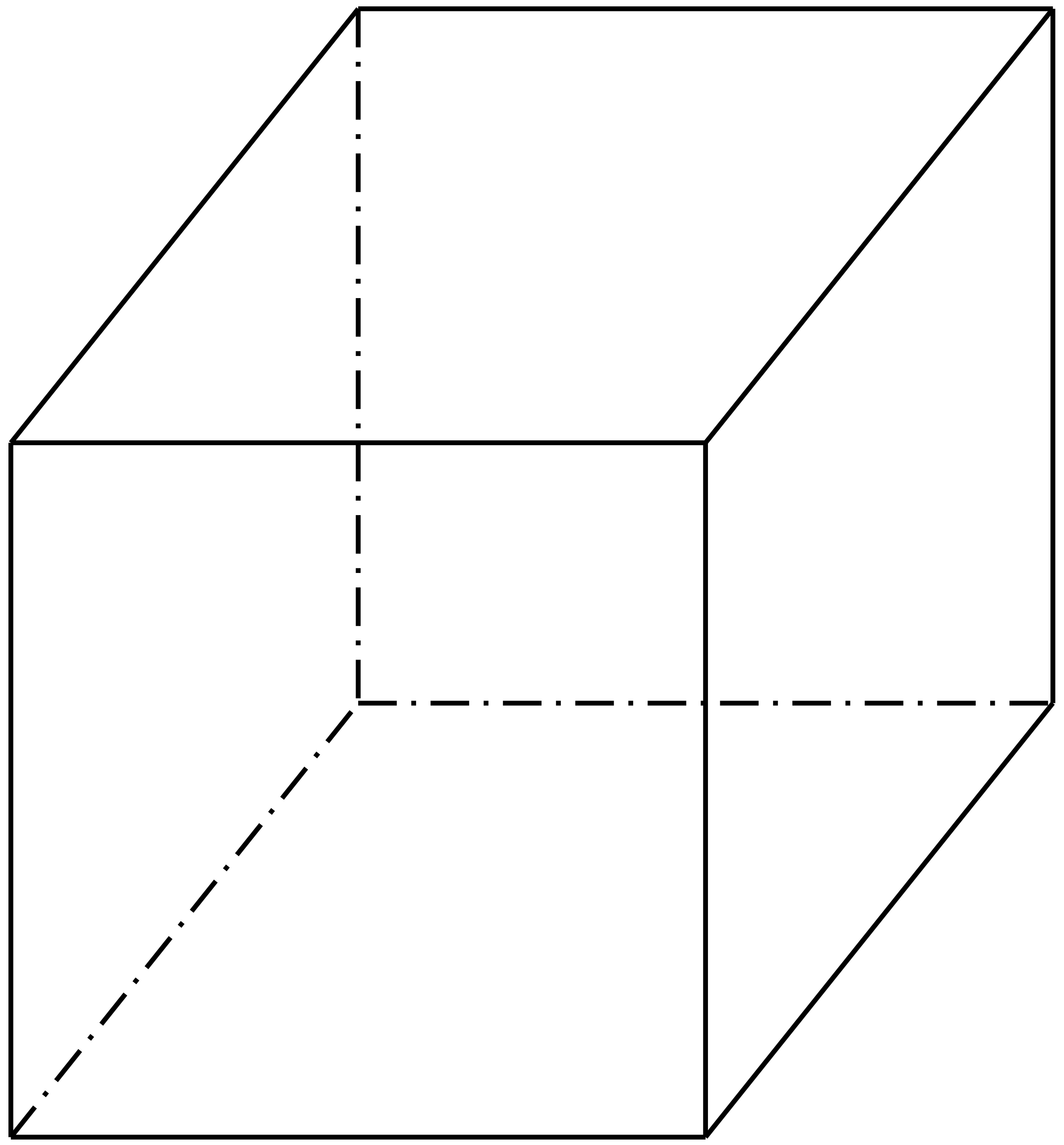
Consider the cube of fluid to the right.

It is composed of an assortment of molecules that are moving, vibrating, etc.

This kinetic energy of the molecules is referred as the internal energy *E*

The internal energy of the cube can change

$$\Delta E = \dots$$

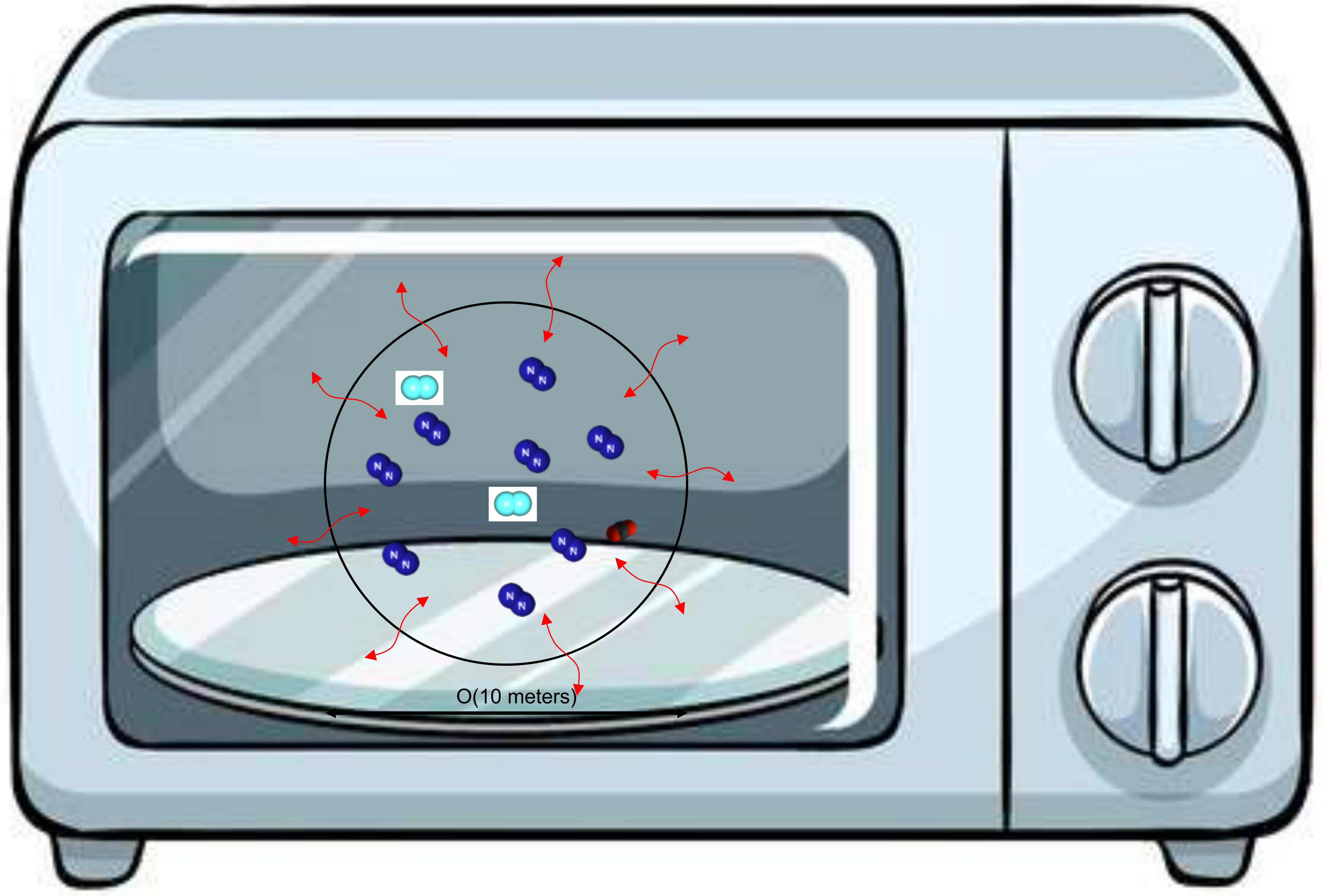


Thermodynamic energy

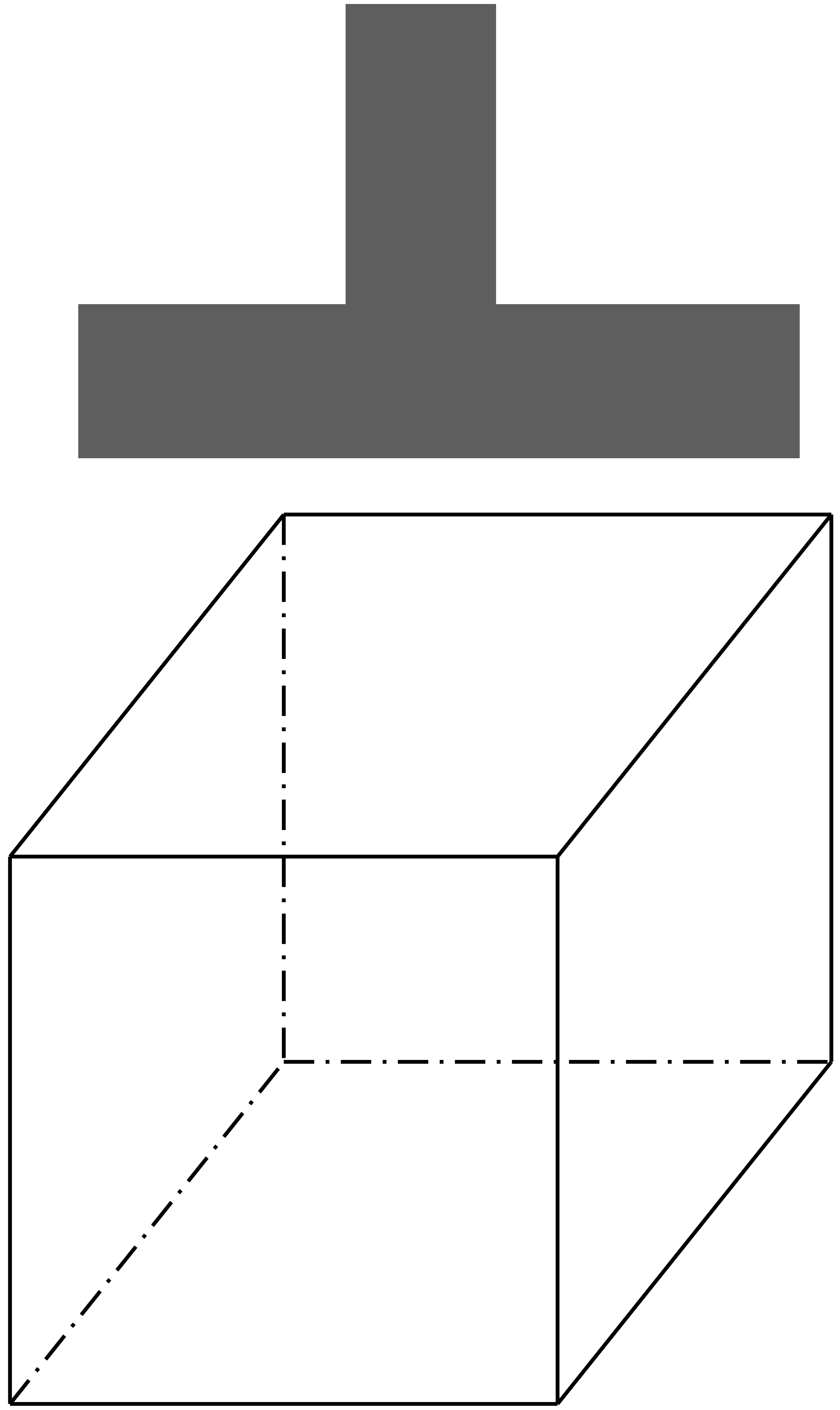
One way to increase its energy is by adding energy in the form of radiation, or through conduction

This is known as **diabatic** heating.

$$\Delta E = Q + \ldots$$



Another way to change the energy of the system is by doing work *W* on it.

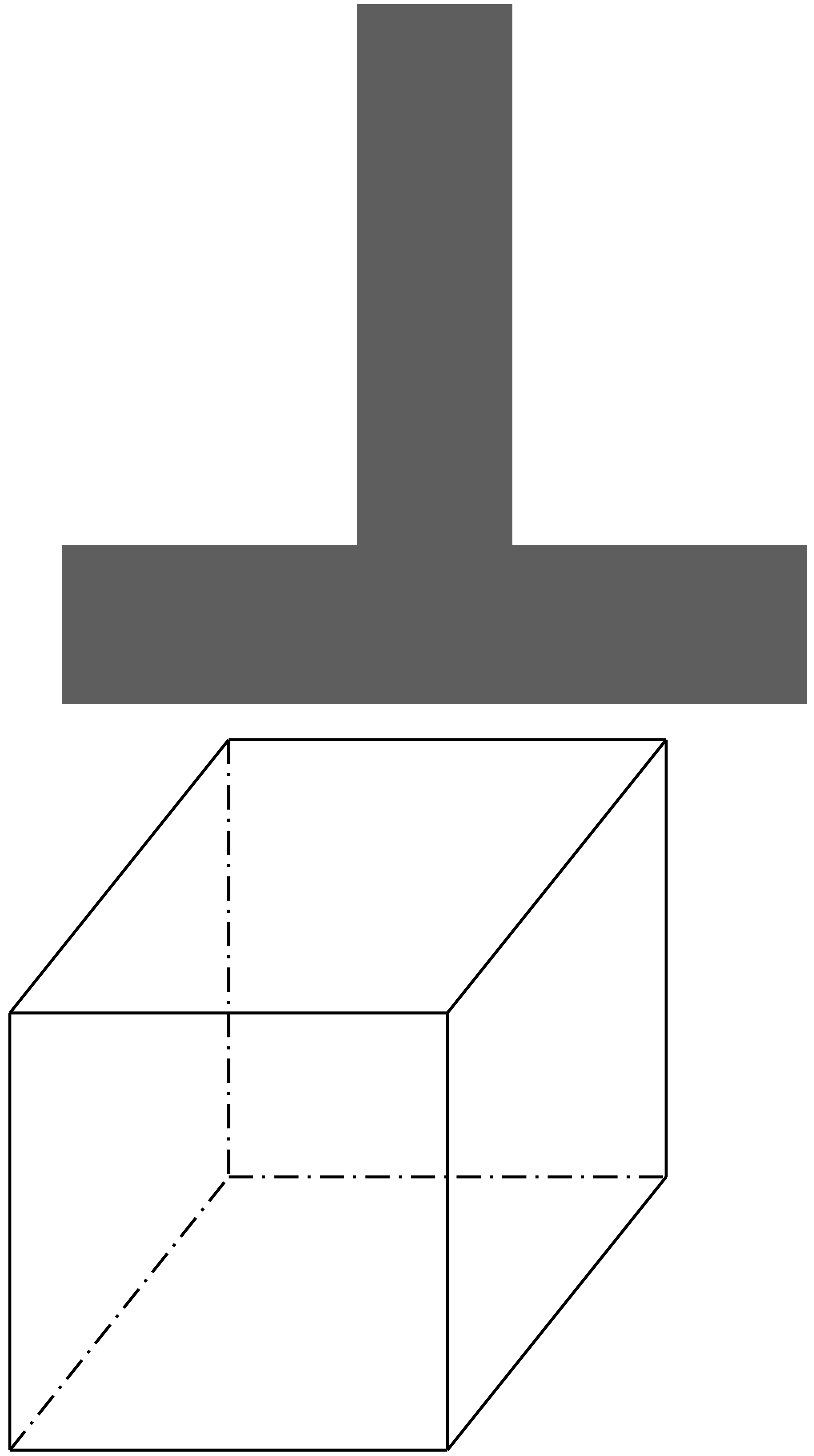


$$\Delta E = \ldots$$

Thermodynamic energy

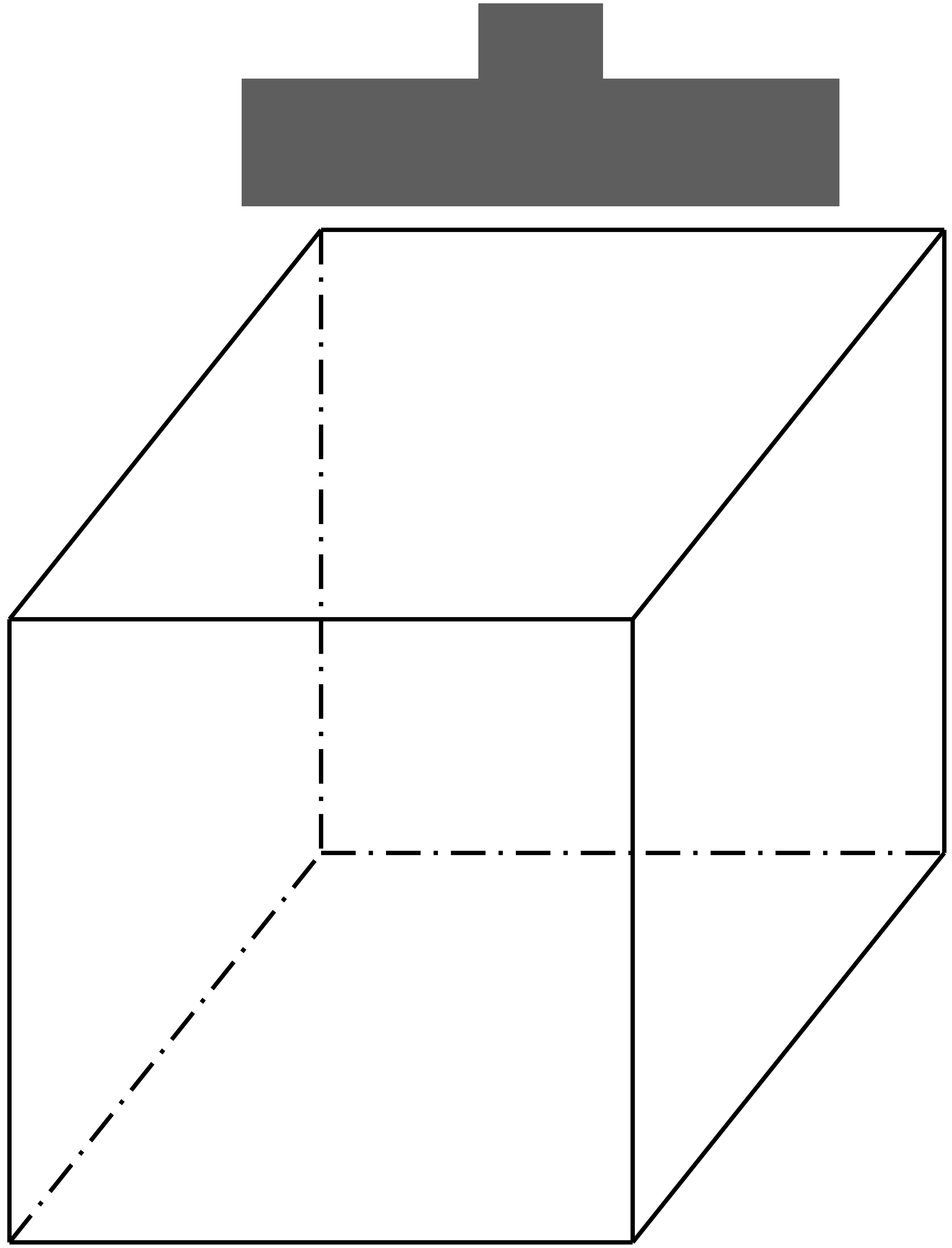
If the piston compresses the cube, then it is doing work on the cube. Energy is transferred from the piston to the cube

$$\Delta E = \ldots$$



If the cube expands and pushes on the piston, then we say that the cube worked on the piston. Energy us transferred from the cube to the piston.

$$\Delta E = \ldots$$

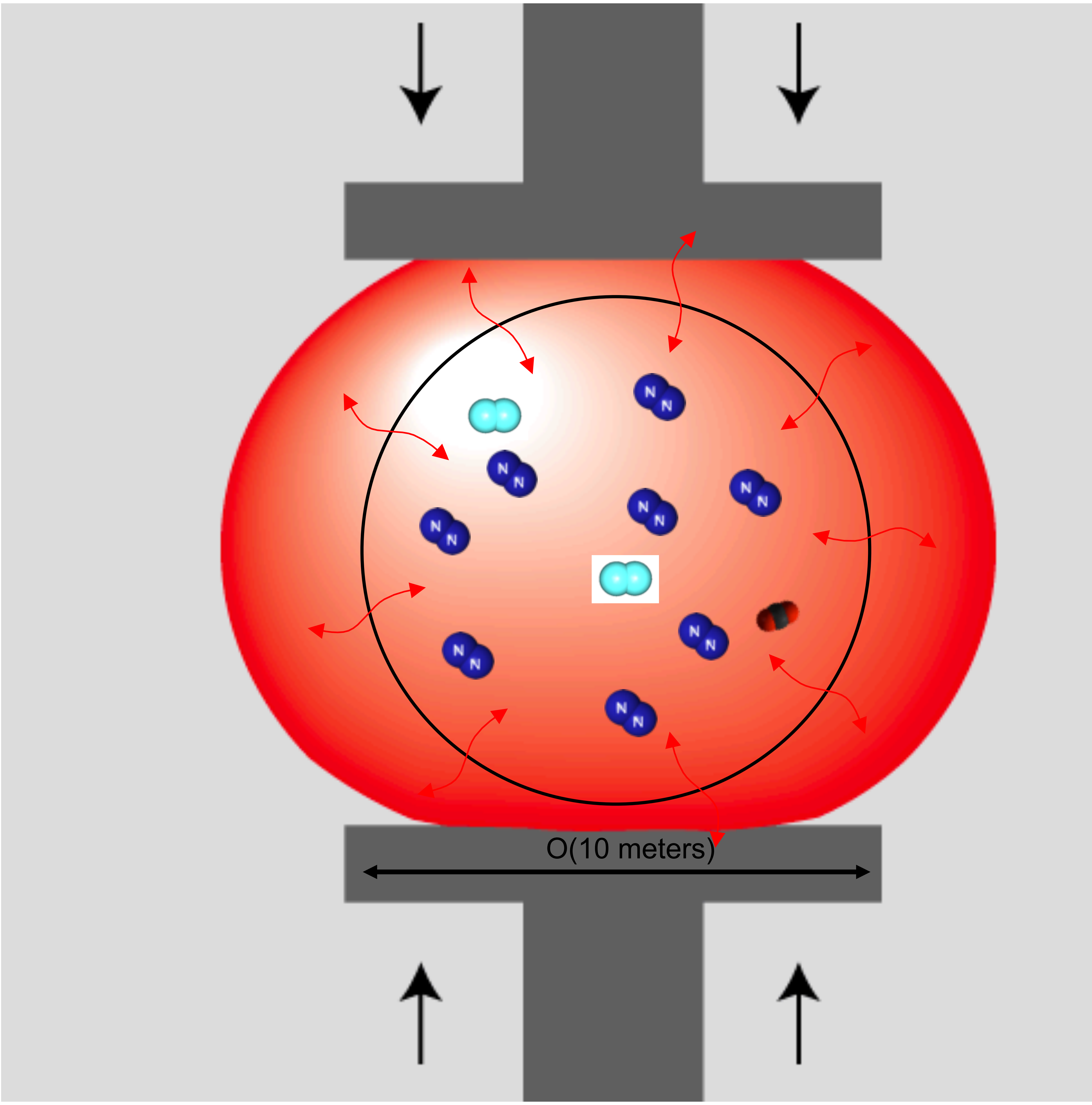


Thermodynamic energy

Think about this in terms of a parcel (the cube is now a parcel).

Imagine the piston is the surrounding environment.

$$\Delta E = \ldots$$

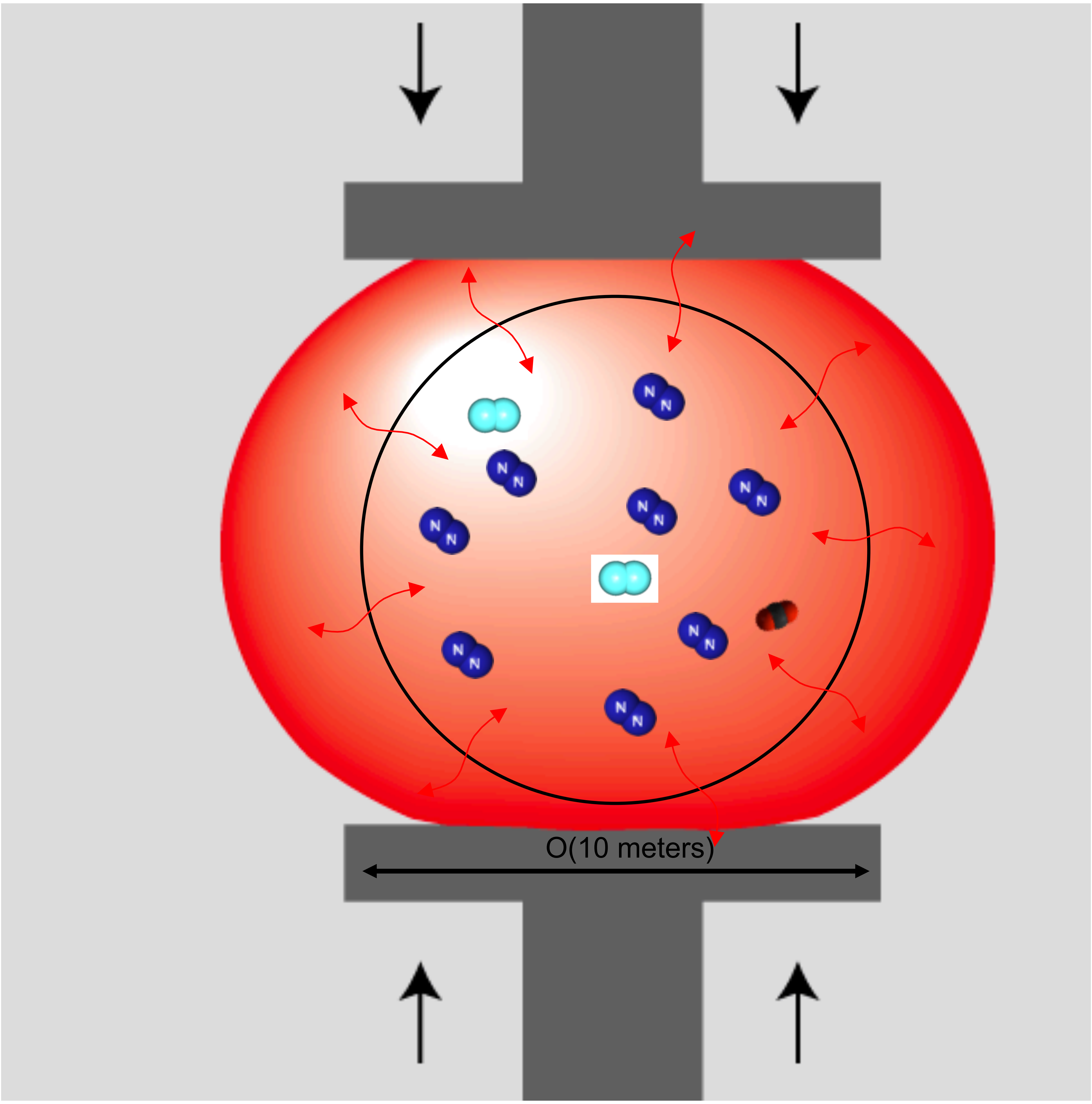


Thus, work can also change the internal energy of the cube

$$\Delta E = Q - W$$

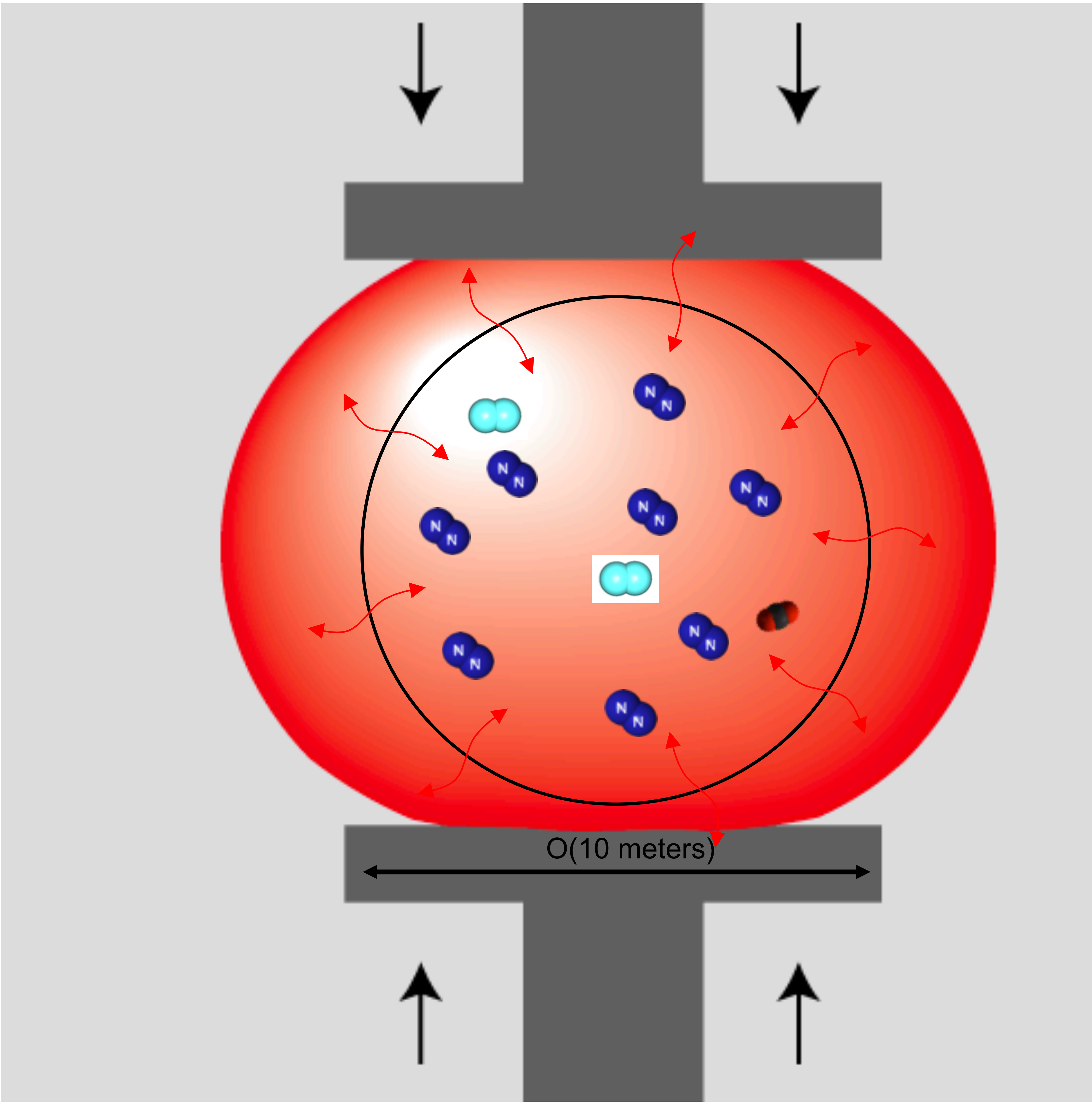
Sign convention is because we express W in terms of the work the parcel does.

Remember: you get tired when you do work, and lose energy



For an infinitesimal increment in energy, we write

$$dE = \delta Q - \delta W$$



Process vs State Variables

$$dE = \delta Q - \delta W$$

$$\Delta T = T_2 - T_1 = \int_{T_1}^{T_2} dT$$

State variables can be written in terms of definitive integrals

$$dT$$

Is the **exact** differential. It satisfies the integral above.

$$Q = \int \delta Q$$

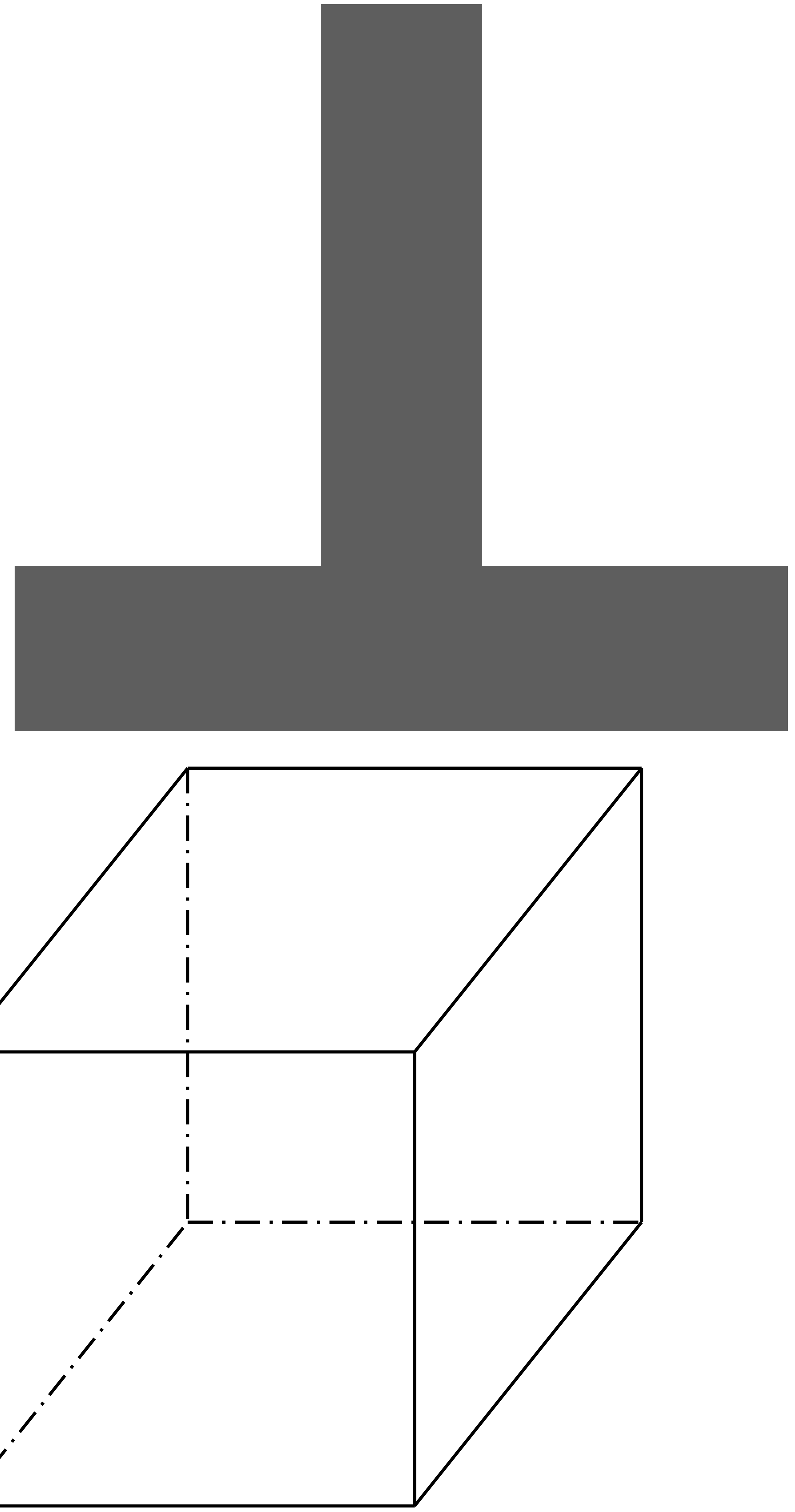
Process variables can only be written in terms of indefinite integrals

$$\delta Q$$

Is the **inexact** differential. It satisfies the integral above.

We can define work as the force applied by the cube times the displacement it causes. In differential form

$$\delta W = F dz$$



Recall that pressure is force per unit area. In this case this is the area of the face where the cube is doing work

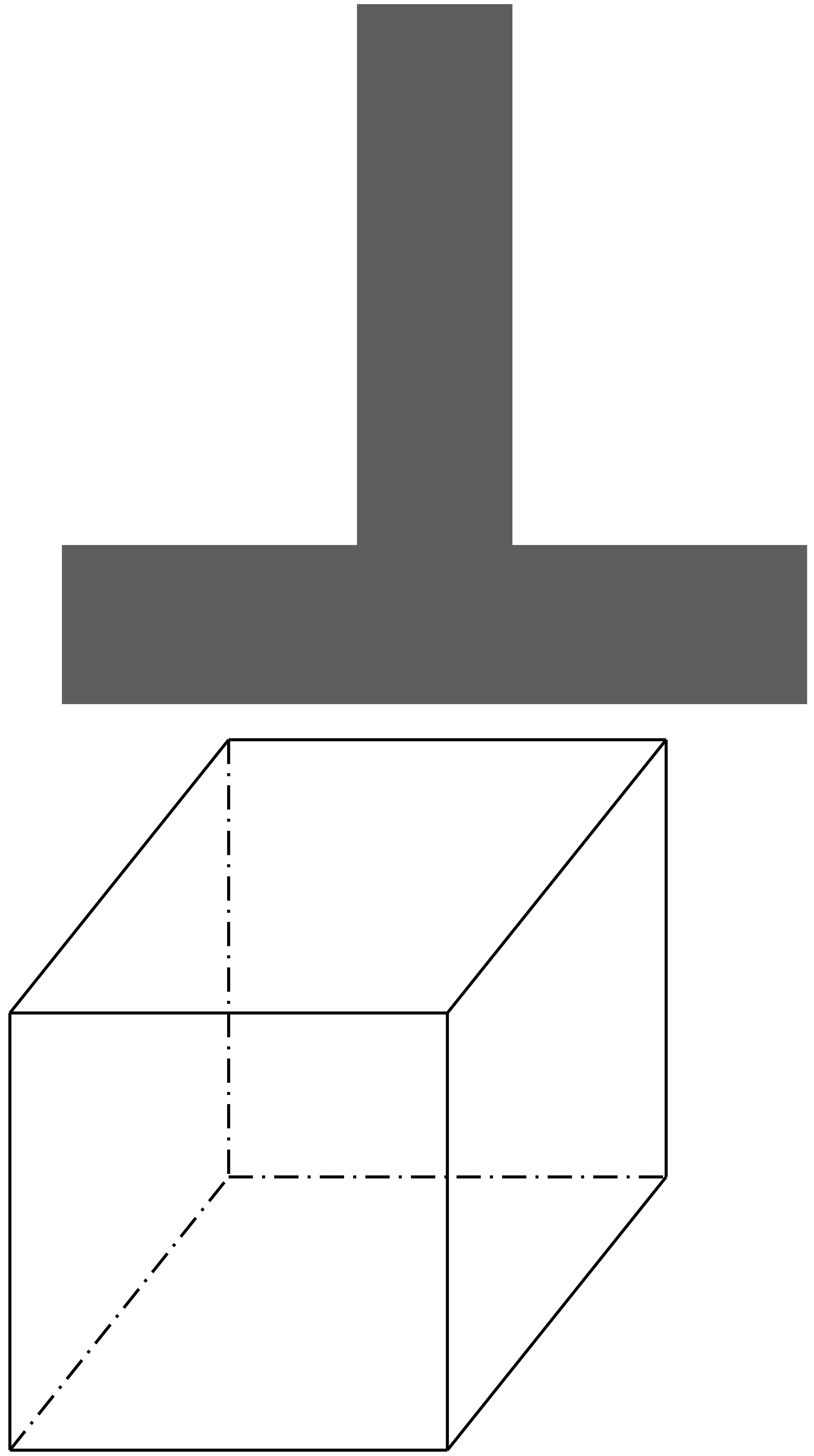
$$\delta W = p dx dy dz$$

Thermodynamic energy

Recall that pressure is force per unit area. In this case this is the area of the face where the cube is doing work

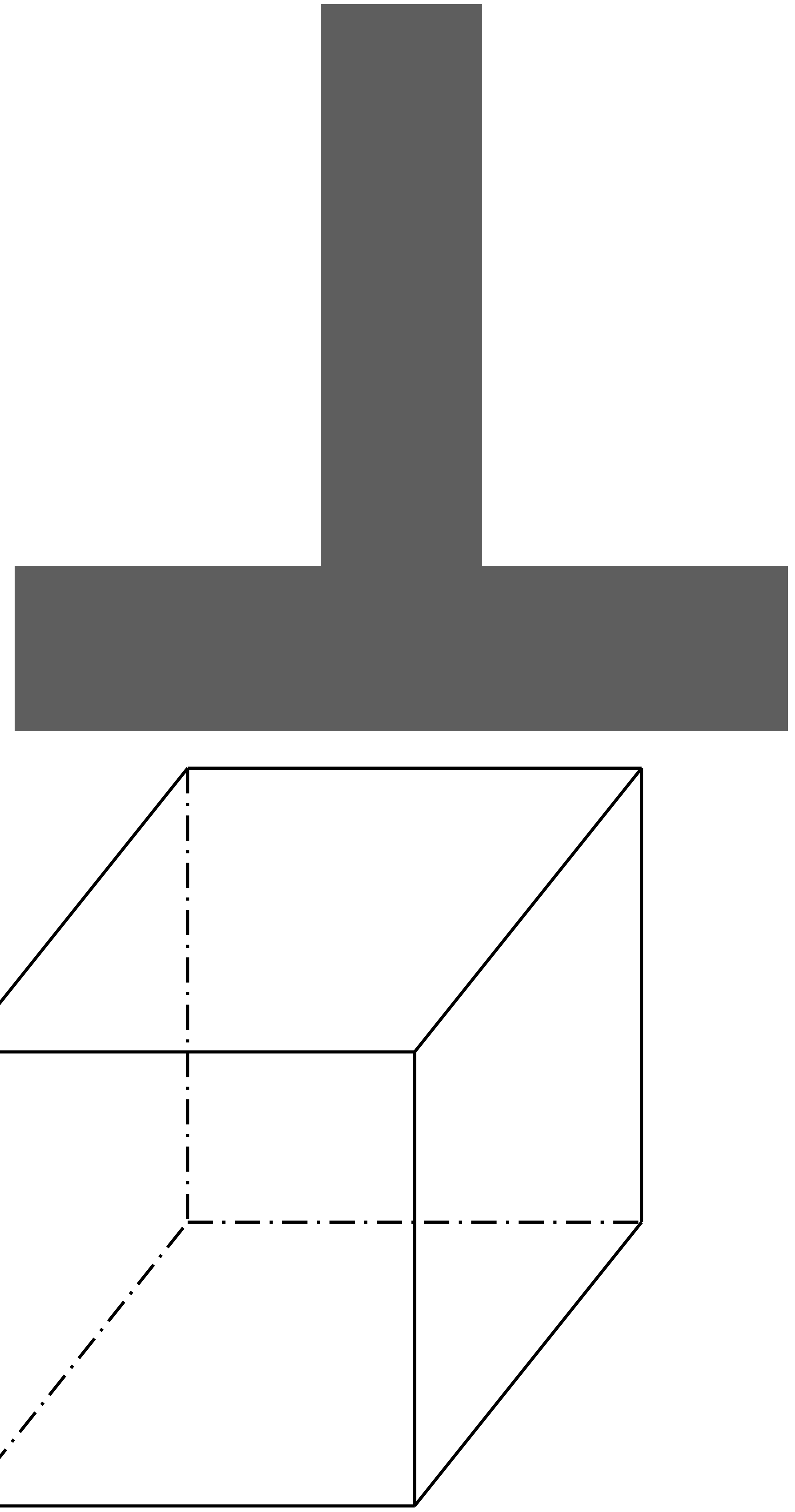
$$\delta W = p dx dy dz$$

The right-hand side is written in terms of exact differentials!



dxdydx is just the volume change *dV* of the cube.

$$\delta W = p dV$$

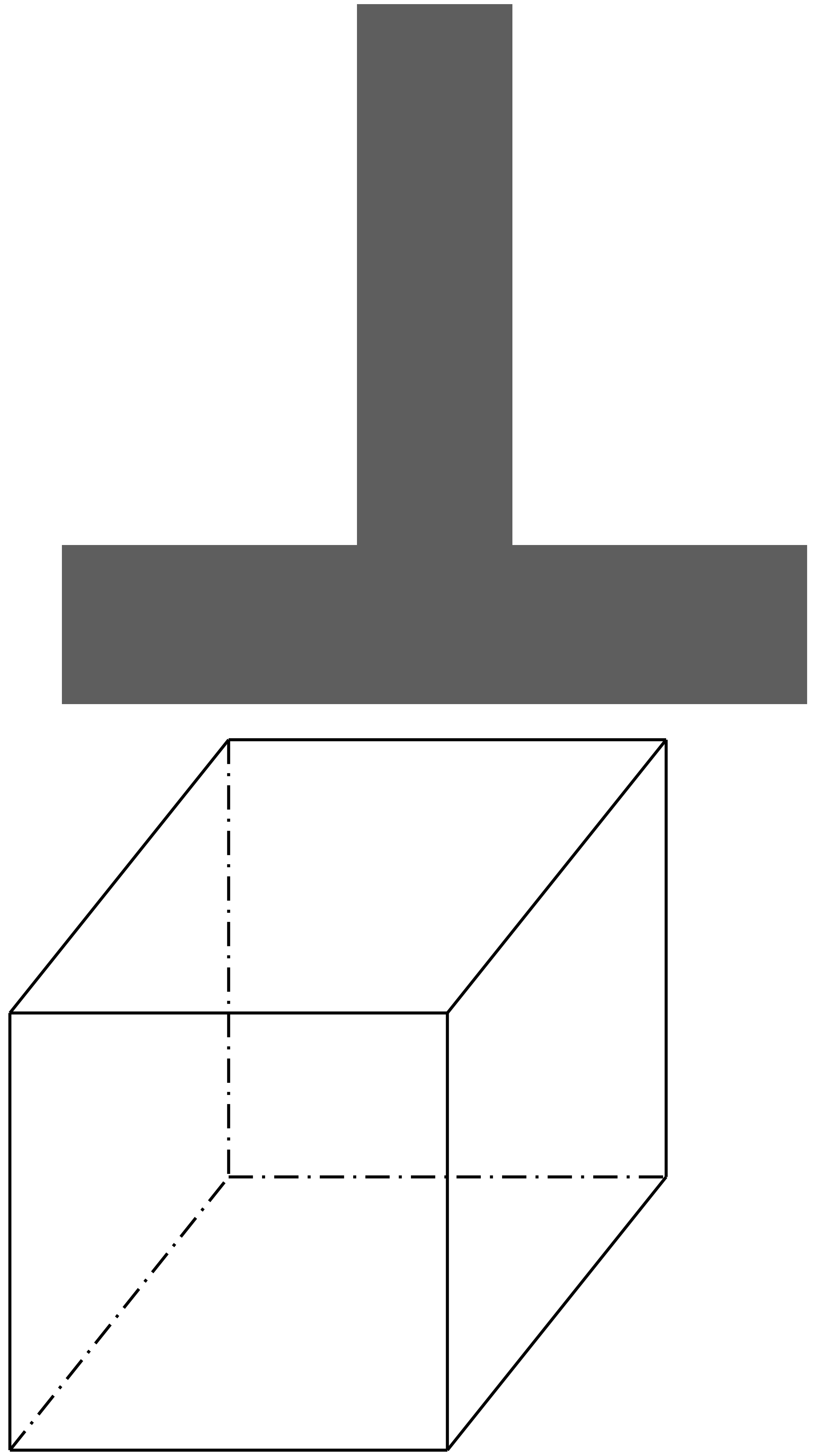


Thus, we can write the total work done as:

$$W = \int \delta W = \int_{V_0}^{V_1} p dV$$

Thermodynamic energy

$$W = \int \delta W = \int_{V_0}^{V_1} p dV$$



We can write this as an intensive variable by dividing by the total mass

$$w = \int \delta w = \int_{\alpha_0}^{\alpha_1} p d\alpha \qquad \alpha = \frac{1}{\rho} = \frac{V}{M}$$

Thermodynamic energy

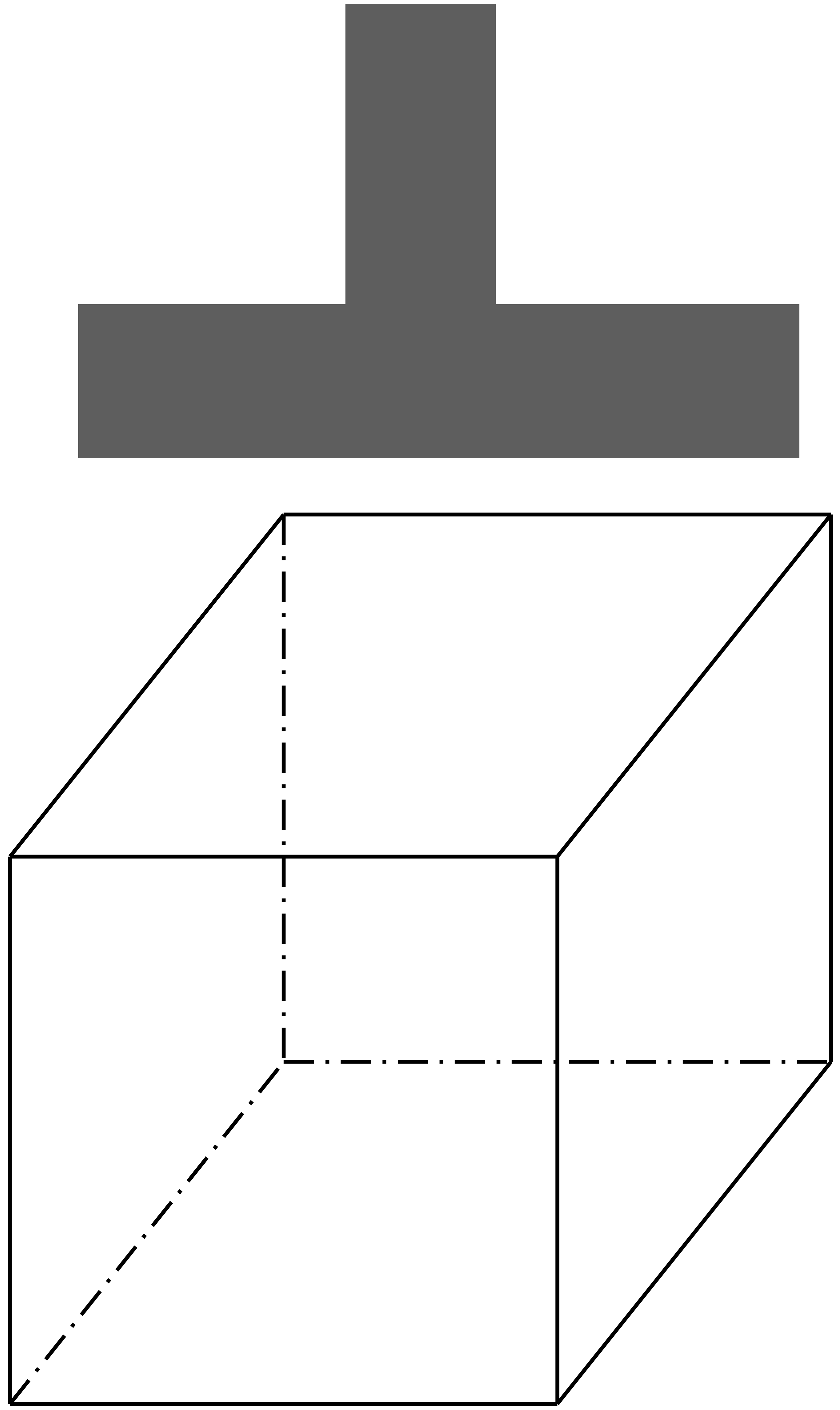
We can divide this equation by the mass of the cube m, to obtain a thermodynamic equation per unit mass (intensive form).

$$de = \delta q - p d\alpha$$

Where the variables are defined as

$$de = dE/M$$

$$\delta q = \delta Q/M$$

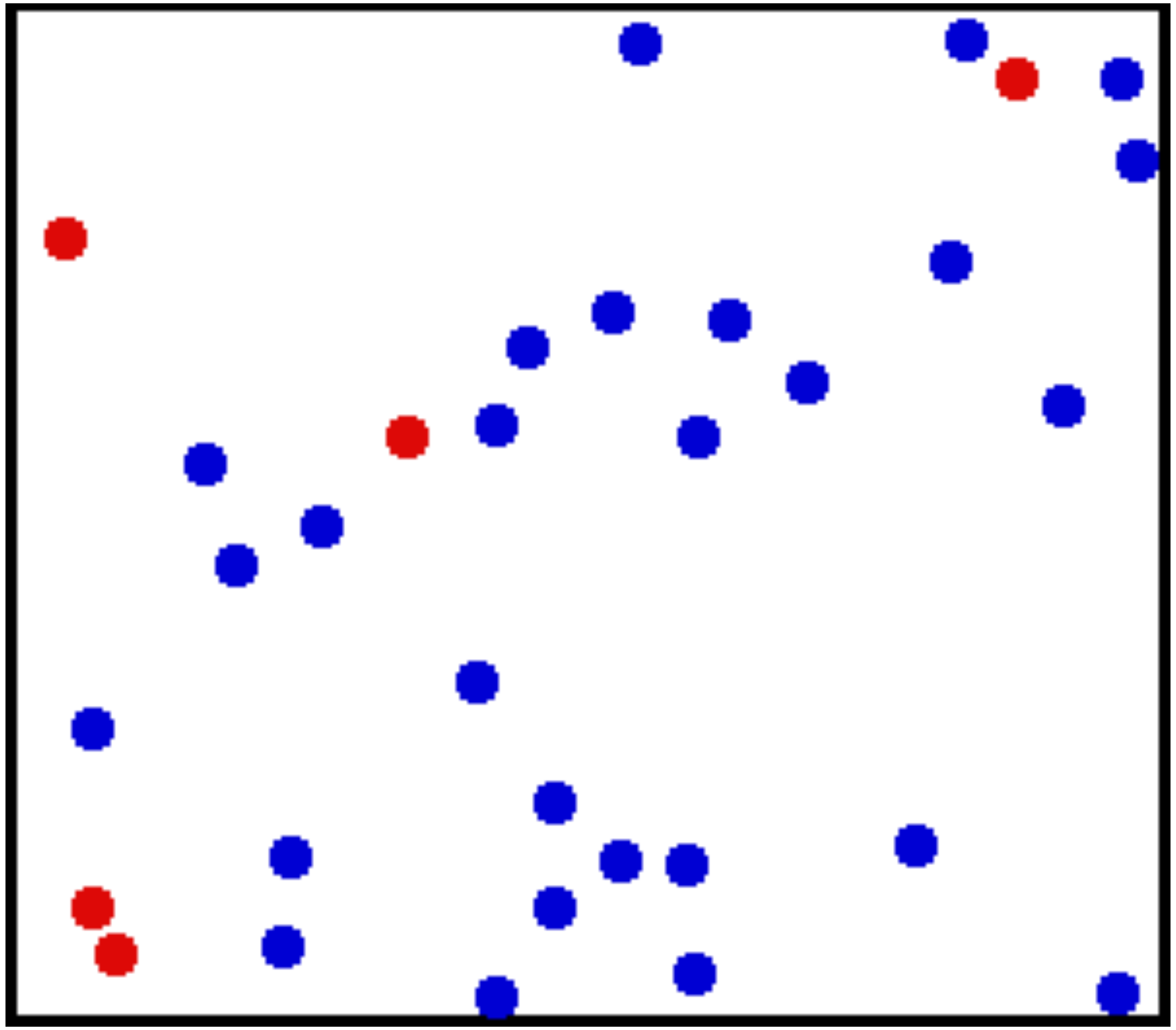


Energy input and temperature

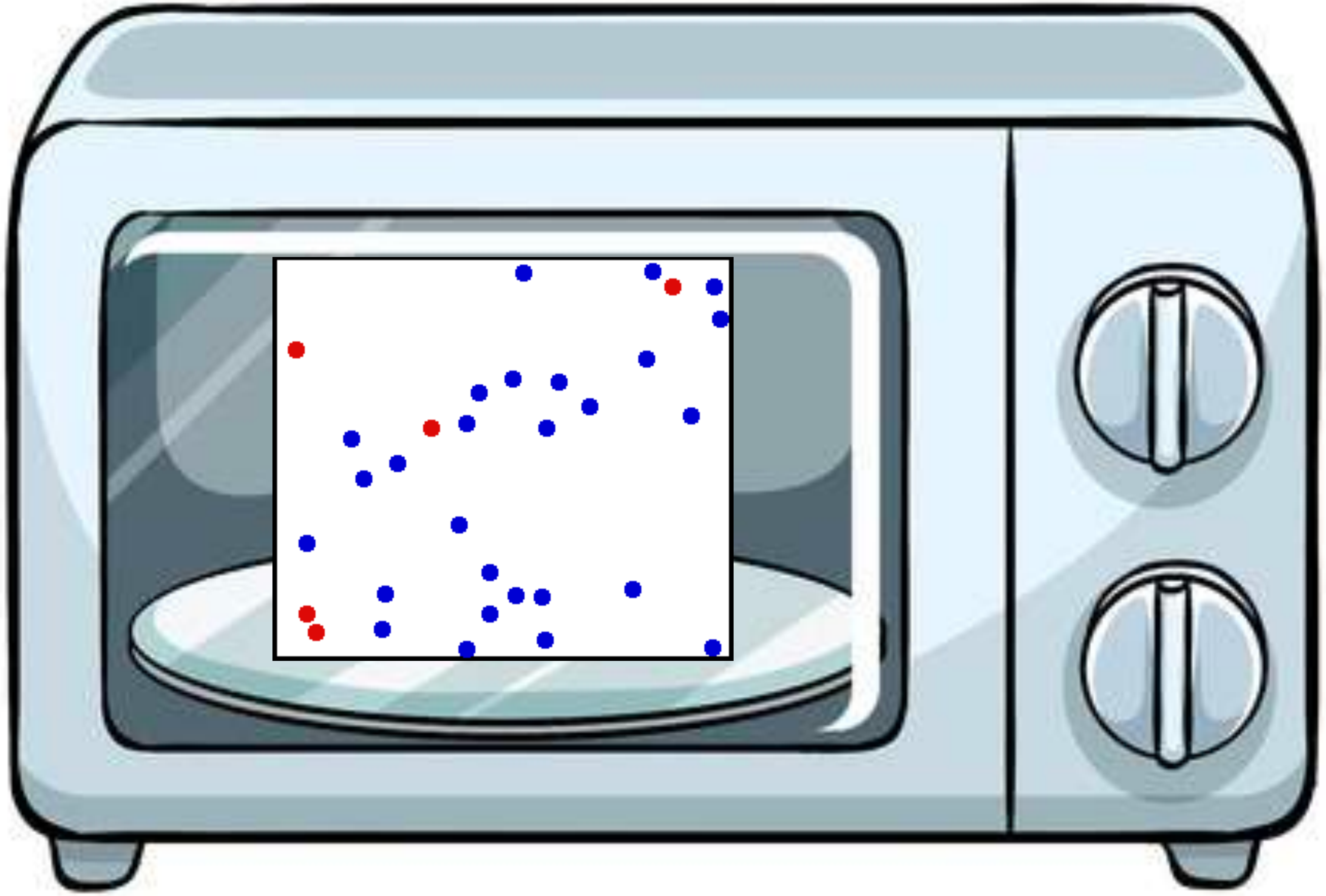
The internal energy is the kinetic energy of the molecules that make up our parcel.

The molecules have more kinetic energy (move faster) when it's hotter.

Internal energy and **temperature** are related.



Energy input and temperature



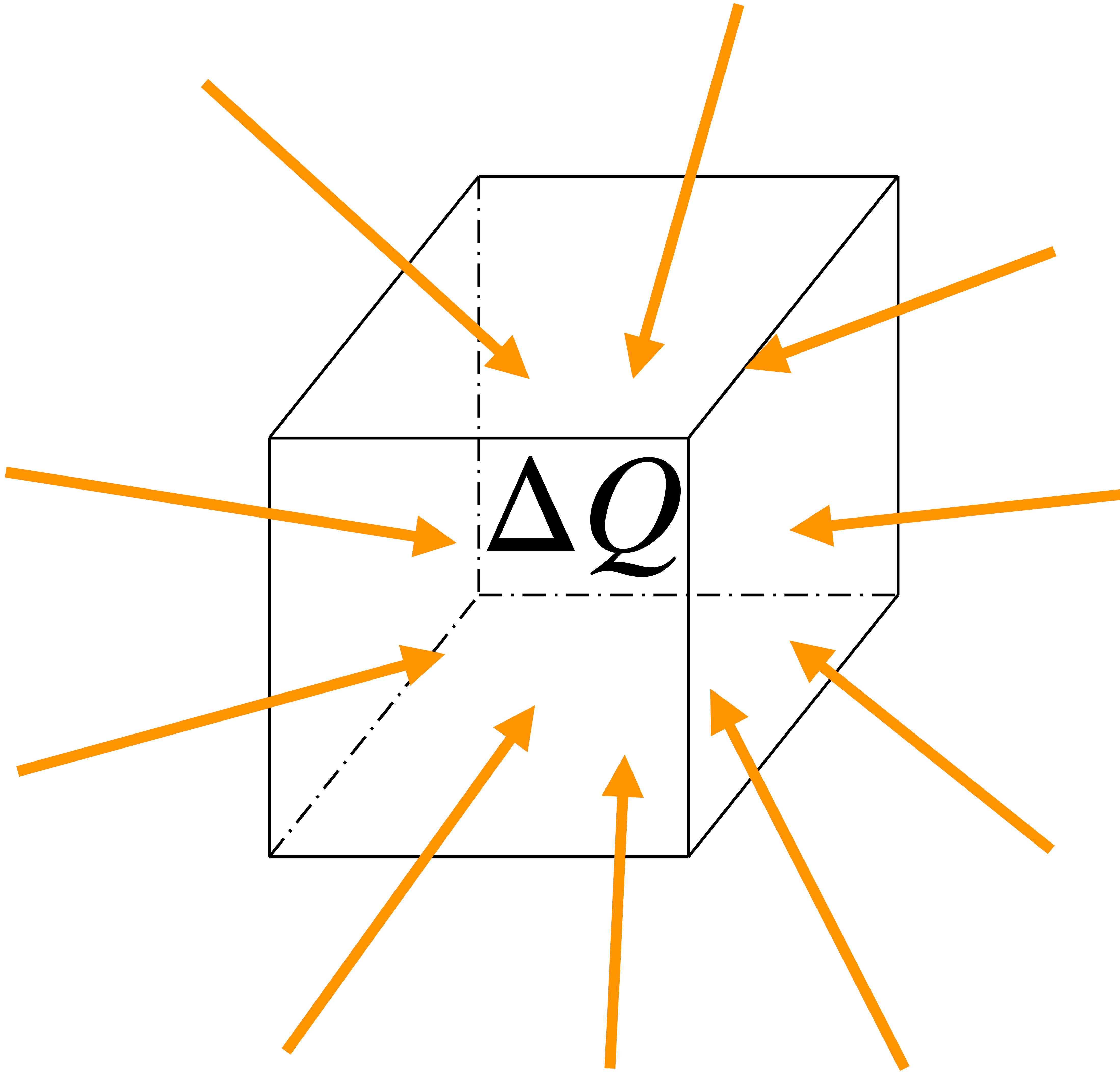
Energy input and temperature

Suppose some energy is input to a cube of fluid. You will notice that its temperature increases.

$$T + dT$$

If we fix the volume constant (no work done), then we can show that the heating is proportional to the change in temperature times a constant

$$dq = c_v dT$$



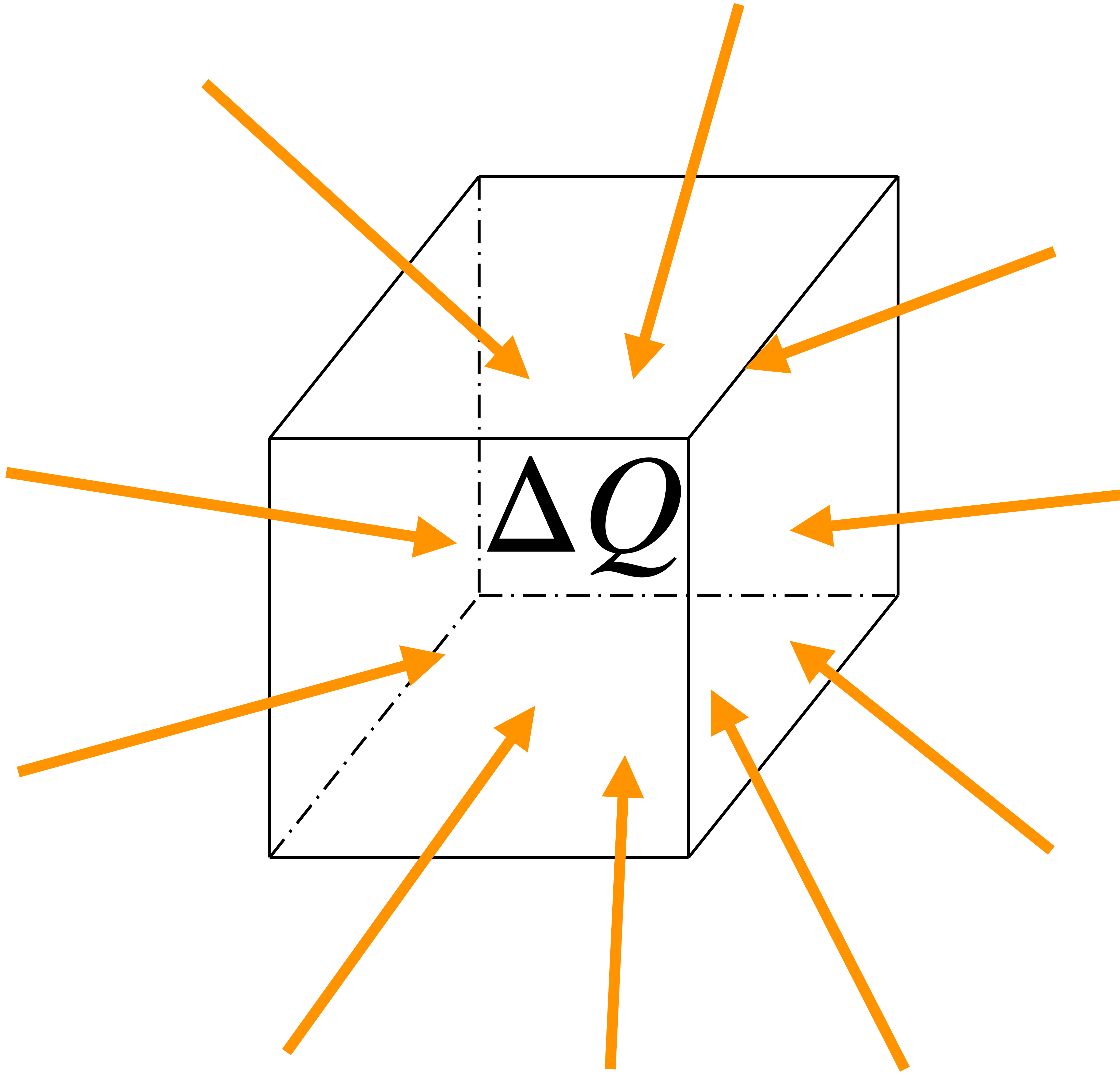
Energy input and temperature

$$\delta q = de = c_v dT$$

c_v is known as the specific heat at constant volume (because volume is fixed)

$$c_v = \left(\frac{de}{dT}\right)_V$$

$c_v = 717\ \mathrm{J\ K^{-1}\ kg^{-1}}$



Thermodynamic energy

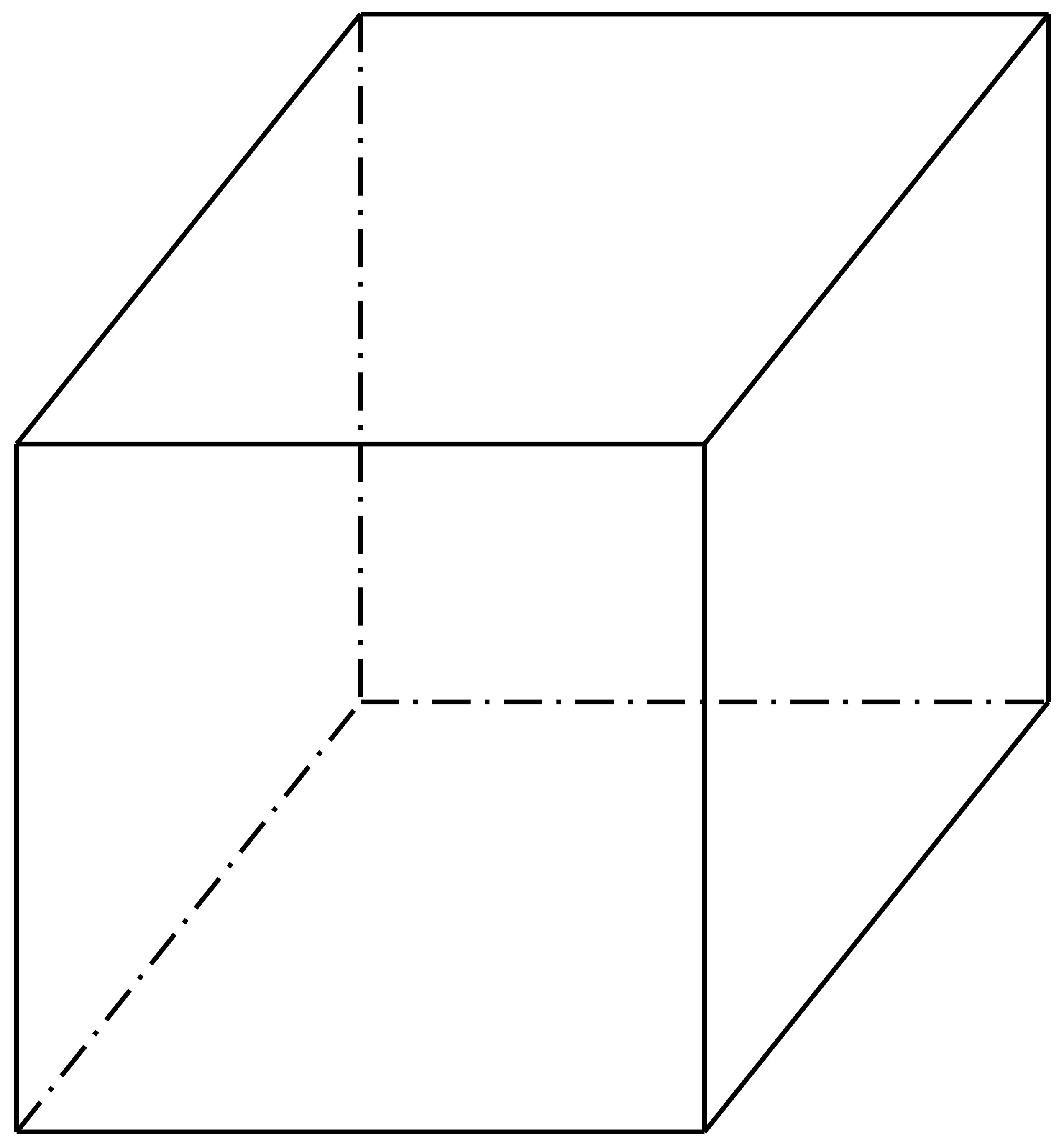
Substituting *de* with $c_v dT$ we get the following

$$c_v dT = \delta q - p d\alpha$$

Where the variables are defined as

$$de = dE/M$$

$$\delta q = \delta Q/M$$



The first law of thermodynamics

Describes the notion of *conservation of energy*.

Also describes the time rate of change of the thermodynamic state

In time-derivative form, we can write as:

$$c_v \frac{dT}{dt} = \dot{Q} - p\frac{d\alpha}{dt}$$

Change in Internal energy

Diabatic heating

Work done by system

$$\dot{Q} \equiv \frac{\delta q}{\delta t}$$

Alternate form for ideal gases

We can use the ideal gas law to write the first law as

$$c_p \frac{dT}{dt} = \dot{Q} + \alpha \frac{dp}{dt}$$

$c_p = c_v + R_d$ Is the specific heat at constant pressure.

We are ignoring the virtual temperature effect.

Types of thermodynamic processes

Isobaric: pressure does not change

Isothermal: temperature does not change

Isochoric: volume does not change

Adiabatic: no heat exchange