The first law of thermodynamics

In differential form: $dE = \delta Q - \delta W$ — in caps b.c. they're written in extensive form.

E = internal energy = state variable

Q and W = process variables
heating, work

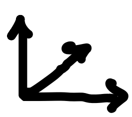
State variables are path independent: $\Delta E = \int_{E_1}^{E_2} dE = E_2 - E_1$

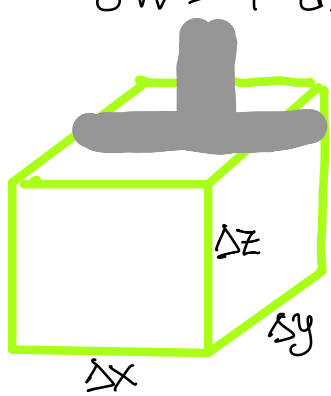
Process variables are path dependent: $W = \int \delta W$ — can't define limit of integration if we don't know the path

What is work: $W = F \Delta Z$

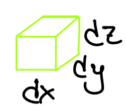
$\delta W = F\, dz$

$F = AP$





for an infinitesimal cube



Force applied to infinitesimal cube

$F = dAp = p\,dx\,dy$

Now we can write work (infinitesimal) as

$$\delta W = F\,dz = p\,dx\,dy\,dz = p\,dV$$

$dV = dx\,dy\,dz$

Back to the first law: $dE = \delta Q - p\,dV$ (extensive form)

Divide by mass (M): $\dfrac{dE}{M} = \dfrac{\delta Q}{M} - p\dfrac{dV}{M}$

Define intensive quantities:

$e = E/M$

$q = Q/M$

$\alpha = V/M$

$\alpha = \dfrac{1}{\rho}$ specific volume

The 1st law in intensive form is: $\boxed{de = \delta q - p\,d\alpha}$

What is the change in internal energy?

$$de = \delta q - p\cancel{d\alpha}$$

If we assume the volume is fixed then...

$$de = \delta q$$

Experiments show that if you keep V fixed

$$\delta q = C_v\,dT \qquad C_v = \left(\frac{de}{dT}\right)_V$$

$$de = C_v\,dT$$

We can rewrite the first law as

$$\boxed{C_v\,dT = \delta q - p\,d\alpha}$$

It may be more intuitive to think about the derivatives in the first law if expressed as changes in time. We can write the first law in time form as:

$$\boxed{C_v\frac{dT}{dt} = \dot{Q} - p\frac{d\alpha}{dt}} \qquad \boxed{\dot{Q} = \frac{\delta q}{\delta t}}$$

is the heating rate ◡

This is the one you want to use in the real world!

Using the ideal gas law $\quad p\alpha = R_d T \rightarrow \alpha = \frac{R_d T}{p}$

$$\frac{d\alpha}{dt} = R_d\frac{d}{dt}\left(\frac{T}{p}\right)$$

$$= \frac{R_d}{p}\frac{dT}{dt} - \frac{R_d T}{p^2}\frac{dp}{dt}$$

$$= \frac{R_d}{p}\frac{dT}{dt} - \frac{\alpha}{p}\frac{dp}{dt}$$

Going back to 1st law $\quad C_v\frac{dT}{dt} = \dot{Q} - R_d\frac{dT}{dt} + \alpha\frac{dp}{dt}$

$\boxed{C_p = C_v + R_d}$ is the specific heat at constant pressure

$$\boxed{C_p\frac{dT}{dt} = \dot{Q} + \alpha\frac{dp}{dt}}$$