## AOS 630: Introduction to Atmospheric and Oceanic Physics Lecture 4 Fall 2021 Thickness and Skew-Ts

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## Announcements

Made some fixes to HW1. It is due a week and a half from now (Thurs next week)

## Last Classs: Hydrostatic Balance

For quiescent atmospheric conditions, the atmosphere is maintained in place by a balance between the downward gravitational force and the upward pressure gradient force.

$$
\rho g \simeq-\frac{\partial p}{\partial z}
$$

Using ideal gas law we can obtain profiles for how pressure changes with height

$$
p \alpha=R_{d} T_{v}
$$

## Hydrostatic Equilibrium



## Hydrostatic balance over the ocean

The ocean equation of state is:

$$
\rho=\rho_{0}\left[1-\beta_{T}\left(T-T_{0}\right)+\beta_{s}\left(S-S_{0}\right)+\beta_{p}\left(p-p_{0}\right)\right]
$$

To a good approximation we can treat seawater as incompresible

$$
\rho \simeq \rho_{0}
$$

We can solve the hydrostatic equation to obtain the following:


$$
p=p_{0}+\rho_{0} g z
$$

## Example

For an isothermal atmosphere, can easily solve the differential equation to obtain

$$
p \simeq p_{0} \exp \left(-\frac{g z}{R_{d} \bar{T}_{v}}\right)
$$

$\bar{T}_{v}$ Mean virtual temperature of troposphere


## Today

Discuss other applications of hydrostatic equation.

Discuss soundings and Skew-T's

# Supplementary reading 

Petty<br>Sections 1.4.3, 1.4.4, 4.2 .5 and 4.3<br>Wallace and Hobbs Section 3.2.2-3.2.4

## Hypsometric equation

For smaller layers of the atmosphere, we can also solve the hydrostatic equation to obtain the "thickness" equation

$$
Z_{2}-Z_{1}=\frac{R_{d}}{g_{0}} \int_{p_{2}}^{p_{1}} T_{v} \frac{d p}{p}
$$

Which we can simplify by replacing the virtual temperature with its layer mean value to obtain

$$
Z_{2}-Z_{1}=\bar{H} \ln \left(\frac{p_{1}}{p_{2}}\right)=\frac{R_{d} \bar{T}_{v}}{g_{0}} \ln \left(\frac{p_{1}}{p_{2}}\right)
$$

Which is known as the hypsometric equation.

## Hypsometric equation

Because of hydrostatic balance, pressure always decreases with height.

Pressure is often used as a vertical coordinate because of this fact.

An isobaric surface is a surface of constant pressure.


## Isobaric surfaces

A map in isobaric coordinates can show the "height" in which the pressure surface occurs.

See HW1, problem 5 to think about why this height can be different.

S

$$
\operatorname{lechex}_{2}
$$

## What about a more realistic atmosphere?

The isothermal atmosphere shows an exponential decrease in pressure with height.

However, temperature decreases quasi-linearly in the atmosphere.

Let's examine a more realistic profile.


## What about a more realistic atmosphere?

Let's assume that the temperature decreases linearly with height

$$
T_{v} \simeq T_{0}-\Gamma z
$$

$\Gamma=\frac{\partial T}{\partial z} \quad \begin{aligned} & \text { is called the lapse rate, assumed } \\ & \text { constant for now. }\end{aligned}$

We can solve the hydrostatic equation to obtain:

$$
p=p_{0}\left(\frac{T_{v}}{T_{0}}\right)^{\frac{g}{R_{d} \Gamma}}
$$



## What about a more realistic atmosphere?

The figure on the right compares the two formulas for some realistic parameters

$$
\begin{gathered}
p=p_{0}=\left(\frac{T}{T_{0}}\right)^{\frac{g}{R_{d} \Gamma}} \quad p \simeq p_{0} \exp \left(-\frac{g z}{R_{d} \bar{T}_{v}}\right) \\
\Gamma=6.5 \mathbf{~ K / k m} \quad H=8 \mathbf{~ k m} \\
T_{0}=300 \mathbf{K} \quad p_{0}=1013 \mathbf{~ h P a}
\end{gathered}
$$

They're pretty similar, right?
Why is that?


## Soundings <br> 年



## Soundings

Radiosondes measure pressure, temperature and humidity.

When a radiosonde is tracked so that winds are estimated, it is called a rawinsonde observation.

Most stations around the world take rawinsonde observations. We just call them all radiosondes cause who cares.


## Soundings

Soundings can be displayed graphically in terms of an emagram.

Temperature is shown in the x -axis (the abscissa).

Pressure is shown in a logarithmic scale in the $y$-axis (the ordinate).

Why is it shown in a log-scale?

## Soundings



## SkewT-LogP diagram

A SkewT-LogP diagram rotates the temperature from being vertical (like the emagram) to being shown as a slanted line.

The rotation is such that a vertical line in the diagram shows a lapse rate of $6.5 \mathrm{~K} / \mathrm{km}$, a typical profile of the atmosphere.


## Comparison



## More on Skew-Ts

As we move forward, we will include more complexity into the Skew-T. A typical SkewT diagram is shown on the right.

Lines include:
Temperature
Dew point (not discussed yet)
Other lines shown
Mixing ratio
Dry adiabatic lapse rate (discussed soon)
Moist adiabatic lapse rate (discussed later)


## Stuves and SkewT

## A Stuve is like an emagram but with more stuff




## Skew-T a week \#1

Download two Skew-T diagrams from two different locations (recommend far away from each other).

1. Discuss the temperature profiles in them.
2. What kind of lapse rate do you see?
3. What can we say about temperature profiles in the atmosphere?

Attach the soundings and discussion of the three points above and upload it to Canvas. It is due one week from today (next Tuesday)

Useful Links:
https://www.aos.wisc.edu/weather/wx_obs/Soundings.html http://weather.uwyo.edu/upperair/sounding.html

