The hypsometric equation:

* Geopotential height

→ Gravity becomes slightly weaker as we increase height

→ Potential energy is usually written as:

$$\Phi^* = M g z \qquad M = \text{mass},\ g = \text{gravity},\ z = \text{height}$$

In intensive form: $\phi = \frac{\Phi^*}{M} = gz$

$$gz = g_0 Z_1$$

g_0: constant Z_1 = geopotential height

g: changes with height

$$d\phi = d(gz) = g_0\, dZ_1 \qquad g_0 = 9.8\ ms^{-2}$$

In defining Z_1, we incorporate changes in gravity into the geopotential height

For most purposes $dz \simeq dZ_1$

* Returning to hydrostatic balance:

$$\frac{\partial P}{\partial z} = -\frac{gP}{R_d T_v}$$

we used ideal gas law

We will integrate for two discrete layers bounded by a pressure P_1 and P_2

P_2

$\overline{T_v}$ (layer-mean T_v)

P_1

$$\frac{1}{P}\frac{\partial P}{\partial Z} = \frac{-g}{R_d T_v}$$

We define $\frac{1}{P} dp = d\ln p$

move T_v to left side

$$\int_{P_1}^{P_2} T_v\, d\ln p = -\frac{1}{R_d}\int_{Z_1}^{Z_2} g\, dz = -\frac{g_0}{R_d}\int_{Z_1}^{Z_2} dZ$$

We can define the layer-mean T_v ($\overline{T_v}$)

$$\overline{T_v} = \frac{\int_{P_1}^{P_2} T_v\, d\ln p}{\int_{P_1}^{P_2} d\ln p} \qquad (1)$$

With Eq. (1), we can obtain the following:

$$\bar{T}_v \ln \frac{p_1}{p_2} = \frac{g_0}{R_d} \underbrace{(Z_{2} - Z_1)}_{\text{thickness } (Z_T)}$$

$$\boxed{Z_T = \frac{R_d}{g_0} \bar{T}_v \ln \frac{p_1}{p_2}}$$