Water vapor and the equation of state:

(1) Mixing ratio: $\boxed{r_v = \frac{M_v}{M_d} = \frac{\rho_v}{\rho_d}}$ * Petty uses $w$ instead of $r_v$

→ mass of water vapor per mass of dry air.
Usually expressed in units of g $H_2O$ / kg dry air

(2) Specific Humidity: $\boxed{q_v = \frac{M_v}{M_d + M_v} = \frac{\rho_v}{\rho}}$

Mass of water vapor per total mass of air

Relationship between $q_v$ and $r_v$

$$\boxed{q_v = \frac{r_v}{1 + r_v}} \quad \text{or} \quad \boxed{r_v = \frac{q_v}{1 - q_v}}$$

given that $(H_2O)_v$ is a trace gas, $r_v \ll 1$; it follows that

$$\boxed{q_v \simeq r_v}$$

Obtain the relation between $r_v$, $q_v$ and $e$

$$r_v = \frac{\rho_v}{\rho_d} \xrightarrow[\text{gas law}]{\text{Using ideal}} \frac{e}{R_v \cancel{T}} \left( \frac{P_d}{R_d \cancel{T}} \right)^{-1}$$

$$r_v = \frac{R_d}{R_v} \frac{e}{P_d} \qquad \text{Define } \varepsilon = \frac{R_d}{R_v} = 0.622$$

$$\boxed{r_v = \varepsilon \frac{e}{P_d}}$$

We know that $r_v \ll 1$ because $P_d \simeq 1000$ hPa, $e \sim 6$ hPa

We have two eqns of state for the atm., one for $(H_2O)_v$, and one for dry air. How can we combine into one eqn. that's simple?

Ideal gas law:

$$\begin{aligned} p &= (\rho_d R_d + \rho_v R_v) T \\ &= \rho R_d \left( \frac{\rho_d}{\rho} + \frac{\rho_v}{\rho} \frac{R_v}{R_d} \right) T \qquad q_v = \frac{\rho_v}{\rho} \\ &= \rho R_d \left( \frac{\rho - \rho_v}{\rho} + \frac{q_v}{\varepsilon} \right) T \qquad \rho = \rho_d + \rho_v \\ p &= \rho R_d \left[ 1 + \left( \frac{1}{\varepsilon} - 1 \right) q_v \right] T \qquad \varepsilon = \frac{R_d}{R_v} \end{aligned}$$

Define the virtual temperature

$$\boxed{\begin{aligned} T_v &= T \left[ 1 + \left( \frac{1}{\varepsilon} - 1 \right) q_v \right] \\ &\simeq T (1 + 0.61 q_v) \end{aligned}}$$

Ideal gas law for moist air is $p = \rho R_d T_v$

---

Hydrostatic balance:

$$\cancel{\frac{Dw}{Dt}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \longrightarrow \frac{\partial p}{\partial z} = -\rho g$$

Let's assume an isothermal atmosphere $T_v = \overline{T_v}$

$$p = \rho R_d \overline{T_v}$$

$$\frac{\partial p}{\partial z} = \frac{-p g}{R_d \overline{T_v}} = \frac{-p}{H}$$

Define $H = \frac{R_d \overline{T_v}}{g}$ scale height

Can rewrite equation as:

$$\frac{1}{p}\frac{\partial p}{\partial z} = \frac{-1}{H} \longrightarrow \int d\ln p = -\int \frac{dz}{H} \qquad \frac{dp}{p} = d\ln p$$

The integral yields a solution of the form:

$$p = p_0 \exp\left(\frac{-z}{H}\right)$$

Hydrostatic balance in the ocean:

$$\rho = \rho_0$$

$$\frac{\partial p}{\partial z} = -\rho_0 g \longrightarrow \int dp = -\rho_0 g \int dz$$

* caution: think about the direction of integration

$$p = p_0 + \rho_0 g z$$

$z$ = depth