AOS 630: Introduction to Atmospheric and Oceanic Physics

Lecture 2   Fall 2021

Thermodynamic Systems and the Equation of State

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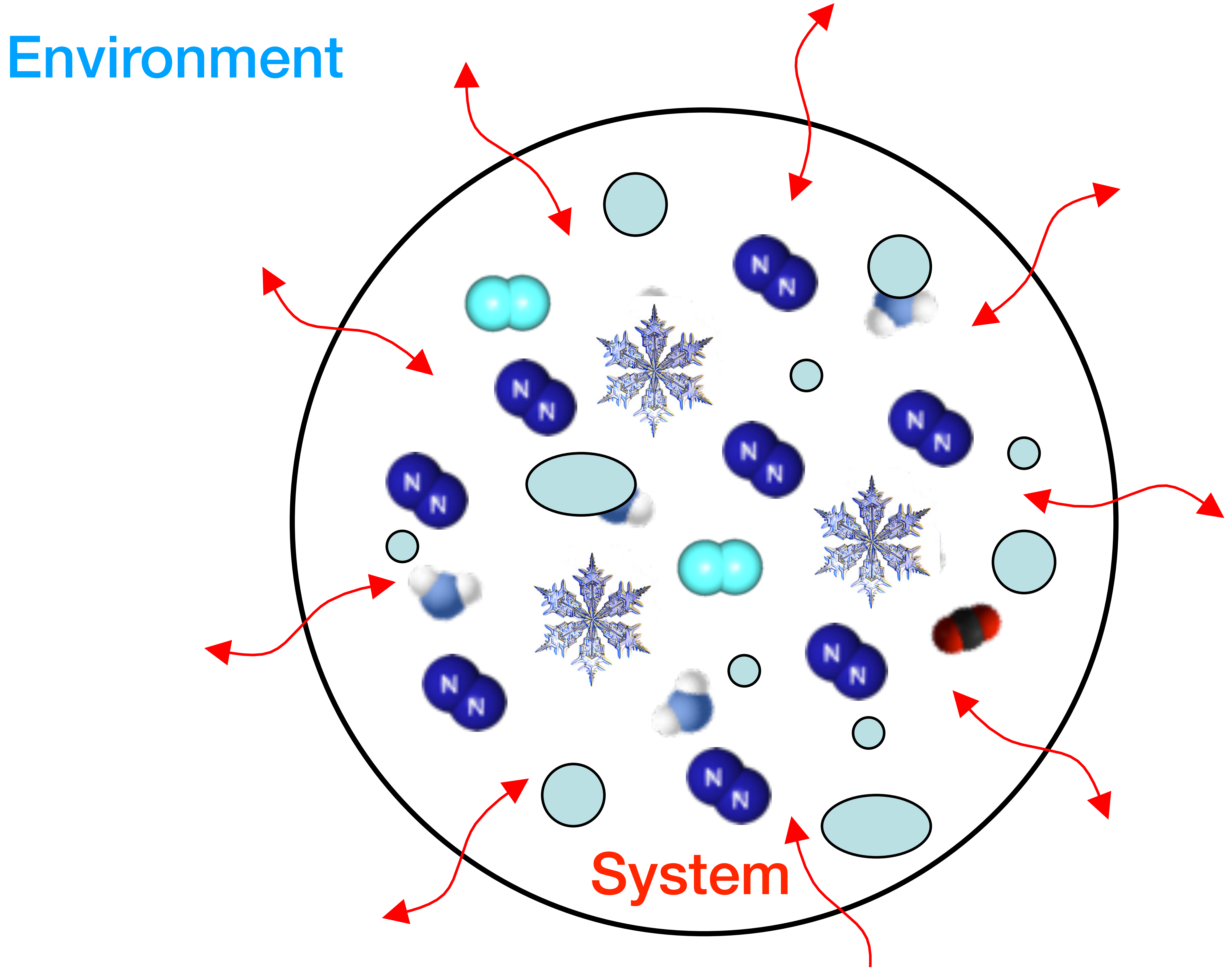
Last class

Concept of a "**parcel**": a volume of air

Large enough to make continuum assumptions (length scale large compared to the molecular mean free path).

Small enough to neglect the effects of (and feedback to) dynamics (e.g., advection).

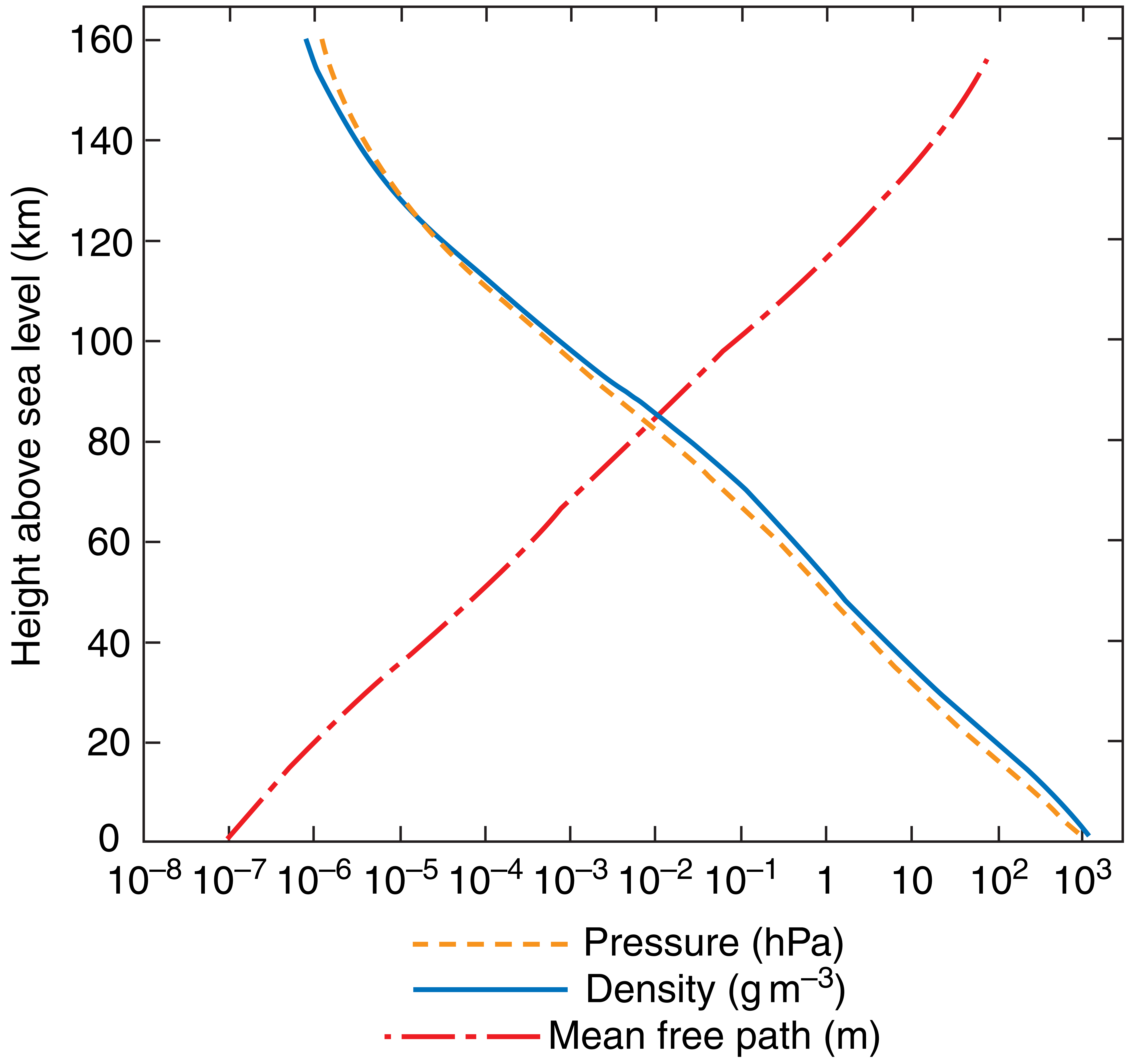
For the purposes of thermodynamics, we consider this a "closed system" (for now).



Atmospheric profile of density and pressure

Density and pressure decrease exponentially with height

(Note the log scale in the figure)



Wallace & Hobbs (2006)

Pressure in the ocean

In contrast, ocean pressure increases linearly with depth.

Parcels of water can also be defined as in air.

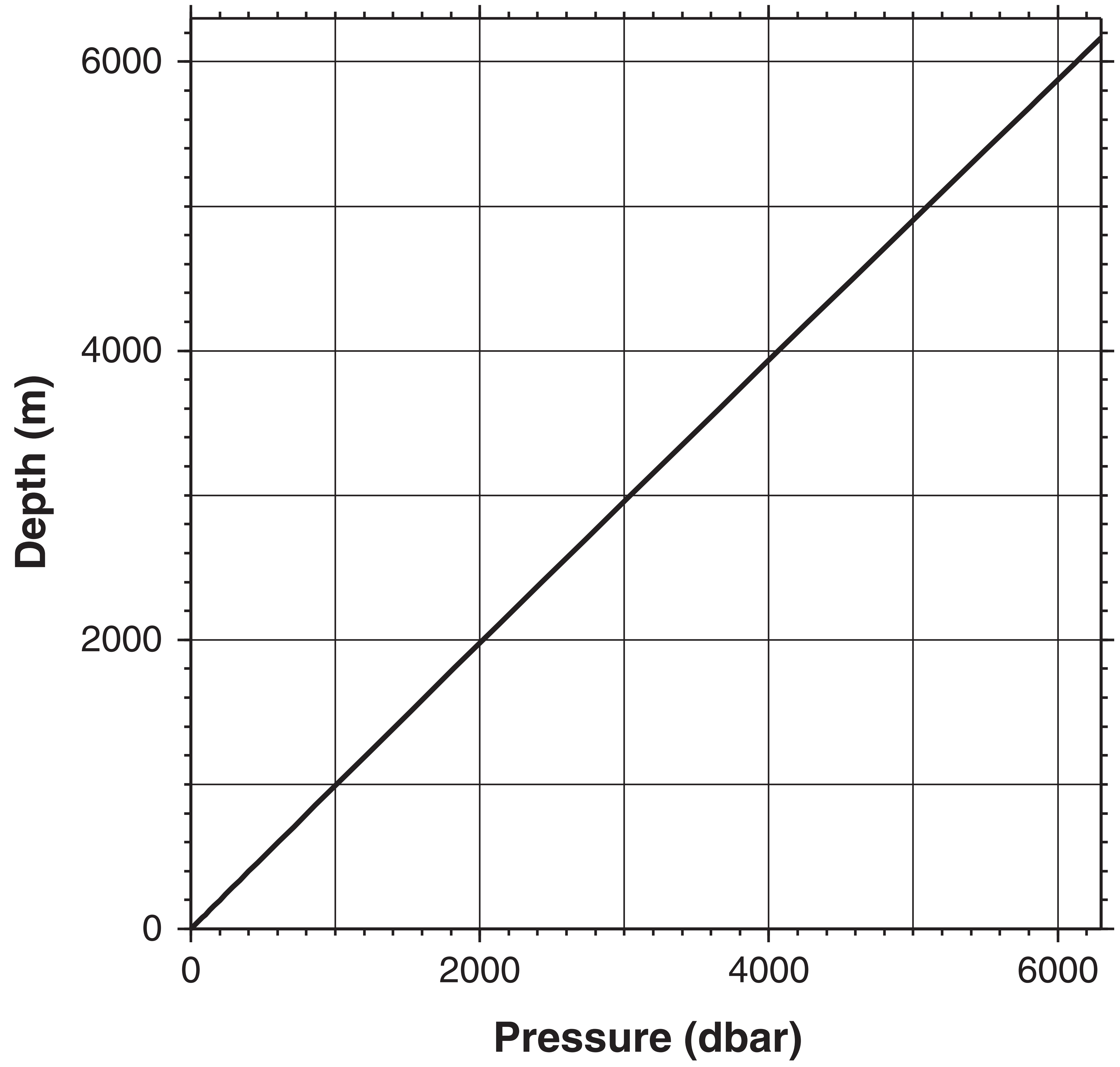


FIGURE 3.2 The relation between depth and pressure, using a station in the northwest Pacific at 41° 53′N, 146° 18′W.

Talley et al. (2008)

Today

Define state variables

Introduce the equation of state

Learning goals

Understand basic definitions of thermodynamics.

Answer the questions:
What is an equation of state?
Why is the equation of state important

Supplementary reading

Petty
Chapters 2 and 3

Wallace and Hobbs
Section 3.1

Ideal_Gas_Law_Derivation.pdf on Canvas

State Variables

A variable that describes the state of a system at any given time

You do **not** need information about the past or future of the system

Their changes are well-defined

State variables: temperature (T), density (ϱ), pressure (p), volume (V)

A variable that describes the transformation of a system between two states

They usually describe a path through time and/or space.

You need information about the past or future of the system

Their changes are not well-defined

Process variables: heating (Q), work (W)

Process vs State Variables

State variables

Only their initial and final values matter, so that

$$\oint_C dT = 0$$

And

$$\Delta T = T_2 - T_1 = \int_{T_1}^{T_2} dT$$

State variables can be written in terms of definitive integrals

Process variables

They describe trajectories and process

$$\oint_C \delta Q \neq 0$$

And

$$Q = \int \delta Q$$

Process variables can only be written in terms of indefinite integrals

Process vs State Variables

$$\Delta T = T_2 - T_1 = \int_{T_1}^{T_2} dT$$

State variables can be written in terms of definitive integrals

$$Q = \int \delta Q$$

Process variables can only be written in terms of indefinite integrals

$$dT$$

Is the **exact** differential. It satisfies the integral above.

$$\delta Q$$

Is the **inexact** differential. It satisfies the integral above.

Extensive vs intensive variables

Extensive variables

Depends on the size of the system.

Examples: Mass, Volume

Intensive variables

Do not depend on the size of the system.

Examples: density, temperature, pressure.

These are preferred in the geosciences.

Equation of State

An equation that describes the relationship between state variables

$$p = f(\rho, T)$$

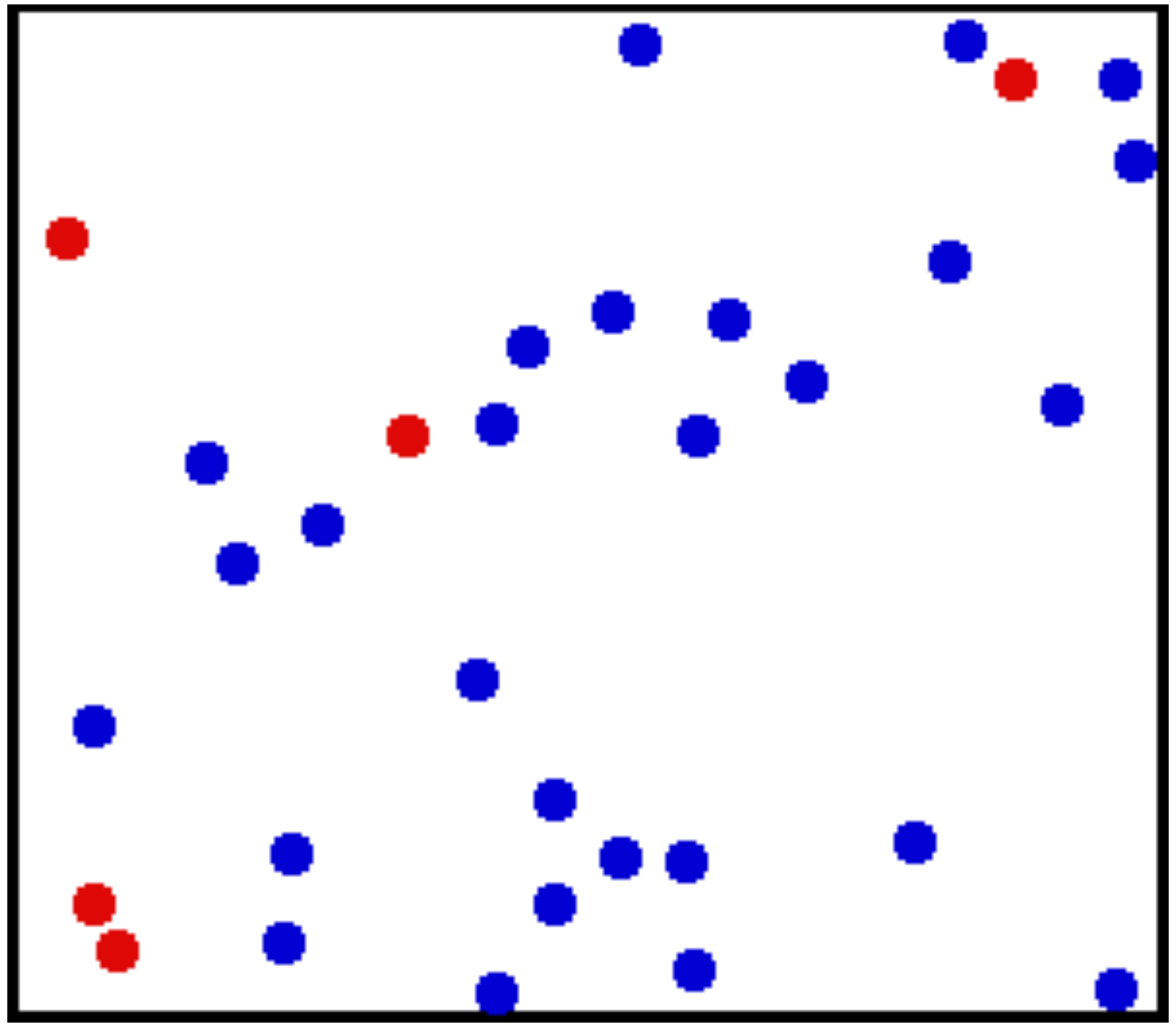
Where *f* means function

Ideal Gases

A mixture of point particles that either don't interact with each other or have perfectly elastic collisions.

To high accuracy, our atmosphere can be described via the use of the ideal gas law

The ideal gas law is the atmosphere's equation of state.



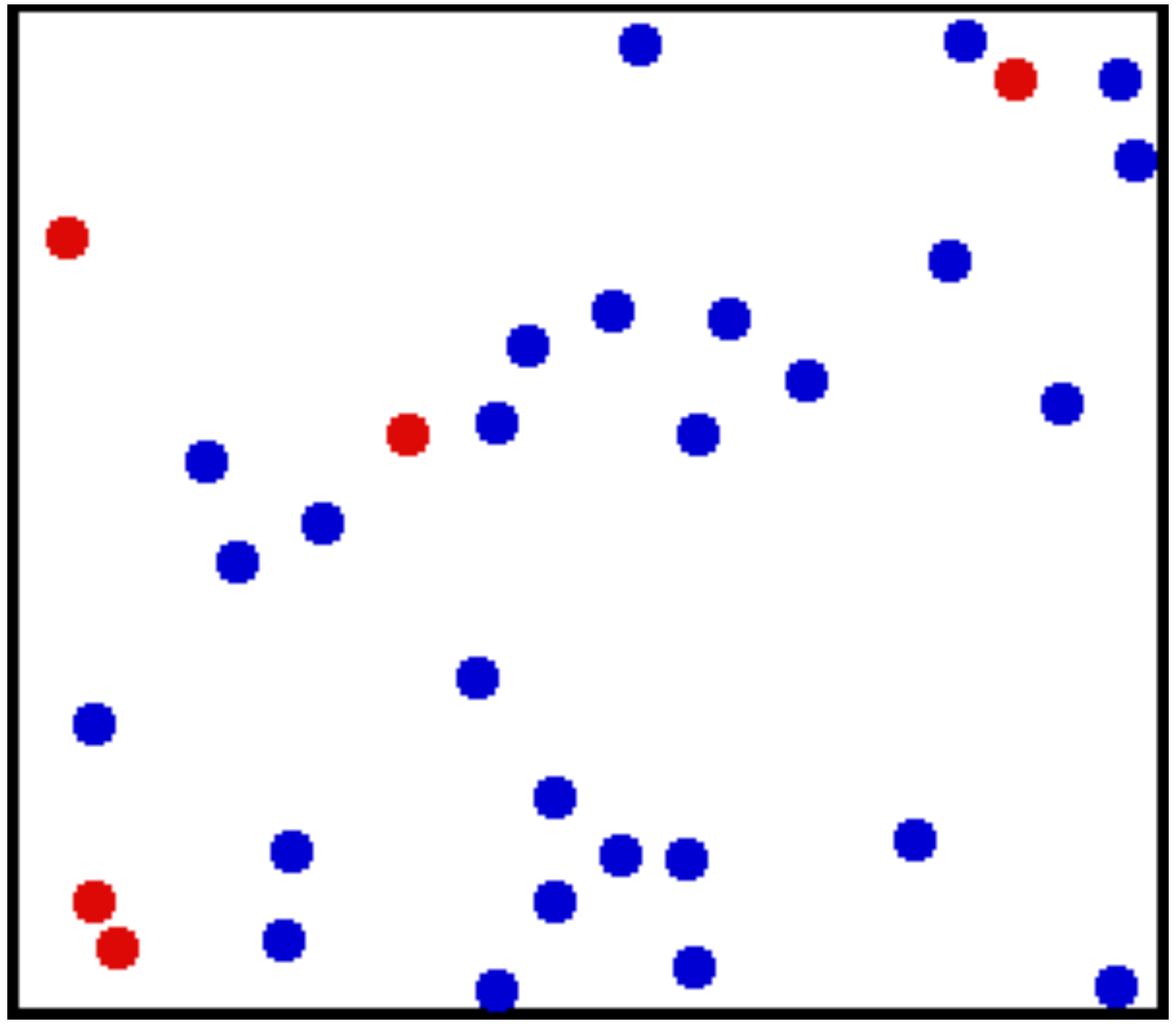
Equation of state for the atmosphere

For dry air

$$p_d = \rho_d R_d T$$

$$R_d = 287 \mathbf{J\ kg}^{-1}\mathbf{K}^{-1}$$

Dry gas constant



Partial pressure for water vapor

$$e = \rho_v R_v T$$

$$R_v = 461 \mathbf{J\ kg}^{-1}\mathbf{K}^{-1}$$

Water vapor constant

Partial pressure for moist air

$$p = p_d + e$$

$$p = (\rho_d R_d + \rho_v R_v) T$$

Equation of state for seawater

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) + \beta_s (S - S_0) + \beta_p (p - p_0) \right]$$

Symbol	Description	Value
ρ_0	reference density	1.027×10^3 kg m^{-3}
α_0	reference specific volume	9.738×10^{-4} m^3 kg^{-1}
T_0	reference temperature	283 K
S_0	reference salinity	35 ppt = 35 g kg^{-1}
c_{s0}	reference sound speed	1490 m s^{-1}
β_T	thermal expansion coefficient	1.67×10^{-4} K^{-1}
β_S	haline contraction coefficient	0.78×10^{-3} ppt^{-1}
β_p	compressibility coefficient ($= \alpha_0 / c_{s0}^2$)	4.39×10^{-10} m s^2 kg^{-1}
c_{p0}	specific heat capacity at const. pressure	3986 J kg^{-1} K^{-1}

This is just an approximation. The equation that is used in ocean modeling is much more complicated!