The equations of state:

i) The ideal gas law:
Often used to describe the gases in our atmosphere.

$$pV = nR^*T$$

p = pressure

V = volume

n = number of kilomoles of gas

R^* = universal gas constant
$= 8314.5 \ \mathrm{J\,K^{-1}\,kmol^{-1}}$

T = temperature

*Works to >99% accuracy in the atmosphere

The form of the ideal gas law that we will use accounts for the different chemicals that exist in our atmosphere.

We assume that the atmos. is a mix of k number of gases

$$n = \sum_{i=1}^{k} n_i$$

n_i is the molar fraction

$$f_i = \frac{n_i}{n} \longrightarrow \sum_{i=1}^{k} f_i = 1$$

The total mass of the system is

$$M = \sum_{i=1}^{k} n_i m_i$$

m_i = molar mass ($\mathrm{kg\,kmol^{-1}}$) or "atomic mass"

Make the ideal gas law based on intensive variables

$$\frac{PV}{M} = \frac{nR^*T}{M}$$

which leads to

$$p\alpha = RT$$

or

$$p = \rho RT$$

$\alpha = \frac{V}{M}$ specific volume

$\alpha = \rho^{-1}$

ρ = density

R = gas constant

*R is not a universal constant. It depends on the specific mix of gases.

$R = \frac{R^*}{\bar{m}}$ $\bar{m}$ is the mean molar mass of a mix of gases

$$\bar{m} = \frac{M}{n} = \frac{1}{n}\sum_{i=1}^{k} n_i m_i \qquad (1)$$

Dalton's law of partial pressures (Petty)

The total pressure exerted by a mix of gases is equal to the sum of the partial pressures that is exerted by each constituent alone if the gas filled the entire volume at the T of the mixture.

$$P = \sum_{i=1}^{k} P_i$$

P_i is the partial pressure

The partial pressure obey the ideal gas law independently

$$P_i = \rho_i R_i T \qquad R_i = \frac{R^*}{m_i}$$

Ideal gas law (eqn. of state) for dry air:

From the last lecture we have:

$N_2 \rightarrow$ 78% of volume composition
$O_2 \rightarrow$ 21%

$$\bar{m}_d = \underset{\text{vol. fraction}}{0.7808}\,\overset{N_2}{\underset{\text{molar mass}}{(28.013)}} + 0.2095\,\overset{O_2}{(31.999)} \qquad \text{Eq (1)}$$

$[\bar{m} = \sum f_i m_i]$

$= 28.9$

Gas constant for dry air: $R_d = \frac{R^*}{\bar{m}_d} = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$

Dry air eqn. of state: $P_d = \rho_d R_d T$

$\rightarrow$ to a good approx. can explain the "state" of air in the atm.

* the equation of state of water vapor

We can use Dalton's law to obtain an ideal gas law for water vapor.

$P_v \longrightarrow e$ water vapor partial pressure

$\rho_v \longrightarrow$ absolute humidity

The ideal gas law for water vapor:

$$\underline{e = \rho_v R_v T} \qquad R_v = \frac{R^*}{m_v}$$

m_v molar mass of H_2O $= 18.016\ kg\ kmol^{-1}$

$$R_v = 461.5\ J\,kg^{-1}K^{-1}$$

Approx. the atmospheric pressure is written as:

$$P = P_d + e \qquad \rho = \rho_d + \rho_v$$

$$P = (\rho_d R_d + \rho_v R_v)T$$

* Water vapor is treated separately b.c. it can change phase*

2.) Equation of state of seawater

* No simple eqn. for it exists

* Seawater is a comb. of H_2O and salts

* It is a liquid